

# NSF Workshop on 21st Century Kinematics

## Computer Aided Invention of Mechanical Movement

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J. Michael McCarthy

Robotics and Automation Laboratory  
University of California, Irvine

ASME IDETC/NSF Workshop on 21st Century Kinematics  
August 11-12 2012  
Chicago, IL, USA



# Introduction

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1. Watt's steam engine and the design of linkages
2. Linkage synthesis theory
3. Why synthesis theory fails and how to fix it
4. Computer aided invention
5. New research

# Welcome



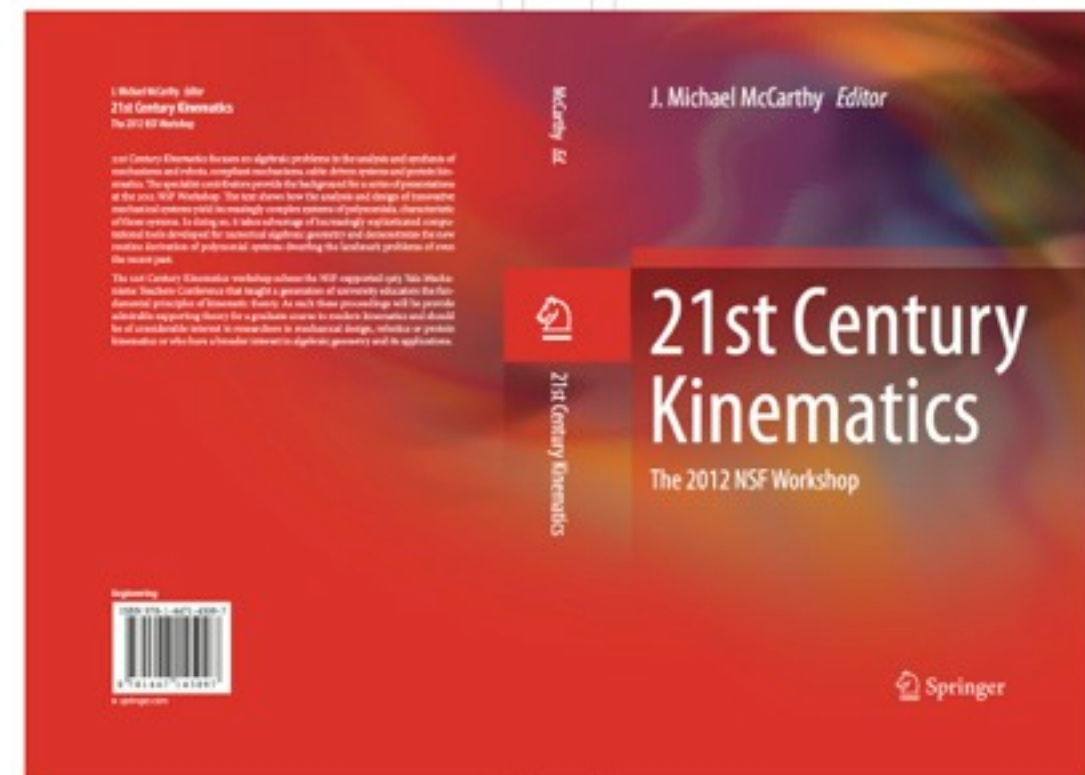
## ASME 2012 IDETC/NSF WORKSHOP 21ST CENTURY KINEMATICS

### SATURDAY, AUGUST 11, 2012

- 12:00n– 1:00pm Registration and Reception
- 1:00 – 1:50pm Computer-Aided Invention of Mechanisms and Robots,  
J. Michael McCarthy, Professor, University of California, Irvine.
- 2:00 – 2:50pm Mechanism Synthesis for Modeling Human Movement,  
Vincenzo Parenti-Castelli, Professor, University of Bologna.
- 3:00 – 3:50pm Algebraic Geometry and Kinematic Synthesis,  
Manfred Husty, Professor, University of Innsbruck.
- 4:00 – 4:50pm Kinematic Synthesis of Compliant Mechanisms,  
Larry Howell, Professor, Brigham Young University.
- 5:00 – 5:30pm Panel Discussion

### SUNDAY, AUGUST 12, 2012

- 8:00 – 9:00am Registration and Reception
- 9:00 – 9:50am Numerical Algebraic Geometry and Kinematics,  
Charles Wampler, Technical Fellow, General Motors Research and Development.
- 10:00 – 10:50am Kinematic Analysis of Cable Robotic Systems,  
Vijay Kumar, Professor, University of Pennsylvania.
- 11:00 – 11:50am Protein Kinematics,  
Kazem Kazerounian, Professor, University of Connecticut.
- 12:00 – 12:30pm Panel Discussion and Closing



Mission:

To present the roots of Kinematics in algebraic geometry and show how computational geometry is changing the opportunities and challenges.



# 20th Century Kinematics

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## In 1912:

- 1875 -- F. Reuleaux: Theoretische kinematik: Grundzüge einer Theorie des Maschinenwesens (Theoretical Kinematics: A foundation for a theory of machines)
- 1888 -- L. Burmester: Lehrbuch der Kinematik
- 1890 -- H. Hollerith's tabulating machine computes the 1890 census in one year.
- 1900 -- Mathematical tables are generated by teams of computers: A computer is a person who calculates.
- 1908 -- Henry Ford introduced the Model T.

## In 2012:

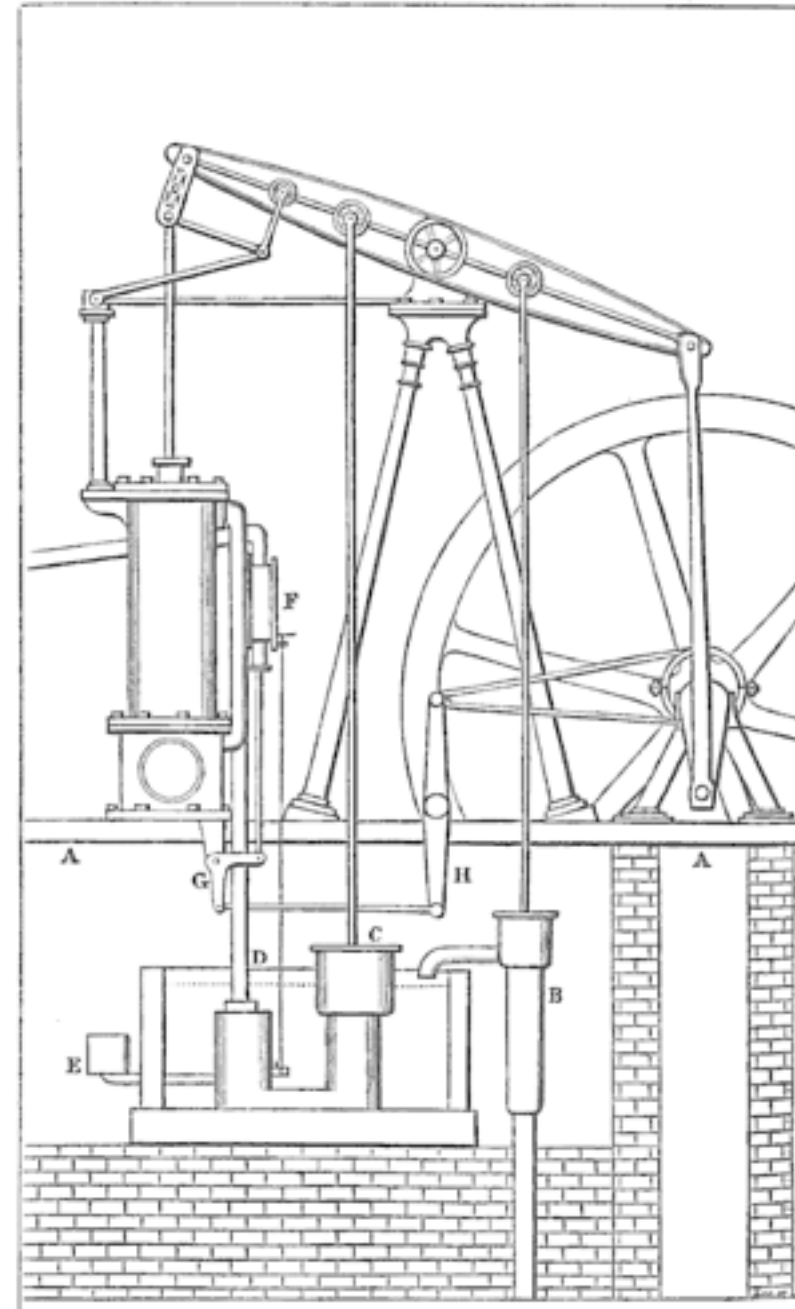
- 1978 -- The PUMA robot is introduced by Unimation based on the design by Vic Scheinman
- 1979 -- O. Bottema and B. Roth: Theoretical Kinematics.
- 1988 -- Qizheng Liao, H. Y. Lee and C. G. Liang reduce the inverse kinematics for the general 6R robot to a single polynomial.
- 1996 -- M. Husty reduces the direct kinematics for the general Stewart-Gough platform to a single polynomial.
- 2007 -- Apple iPhone is introduced.



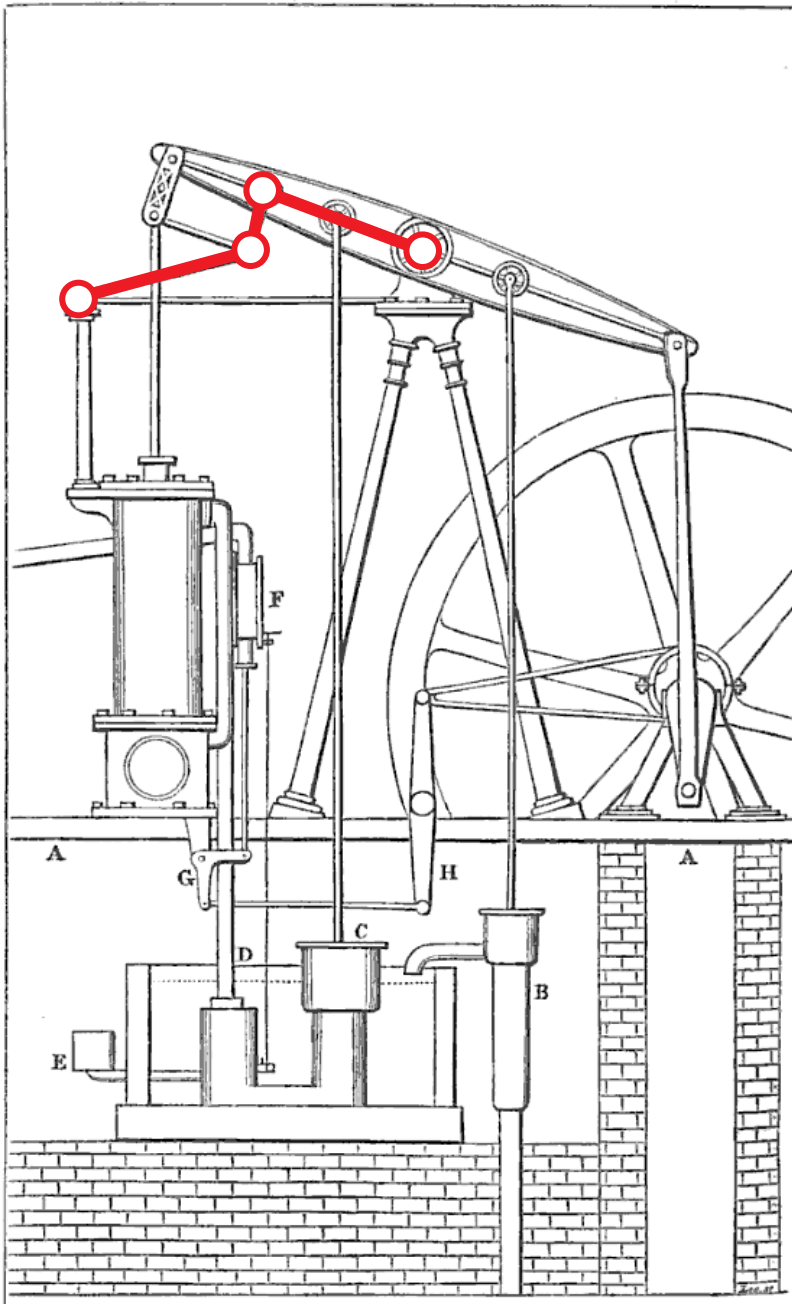
# The Steam Engine

## Boulton & Watt Steam Engine

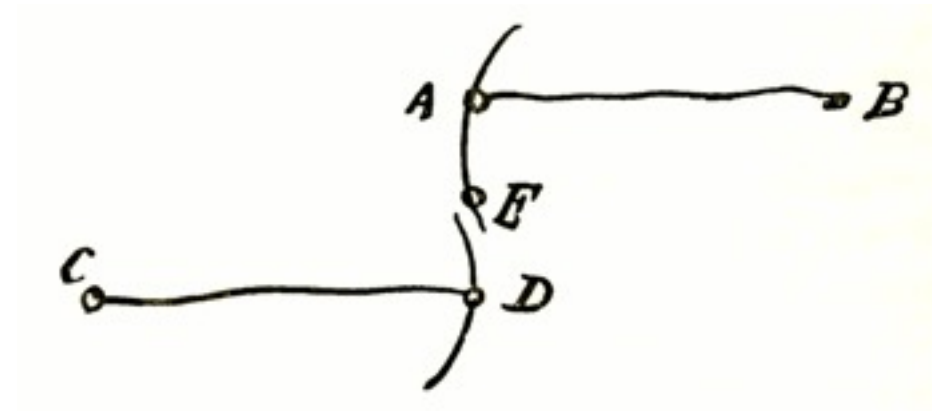
- Steam expansion chamber
- Separate condenser
- Walking beam
- Epicyclic gear drive
- Flyball governor
- Watt linkage



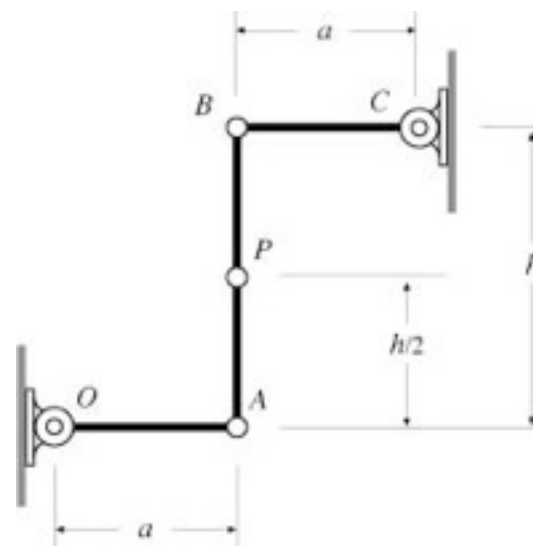
# Watt's Straight-line Linkage



Watt's straight-line linkage



James Watt: "Though I am not over anxious after fame, yet I am more proud of the parallel motion than of any other mechanical invention I have ever made."

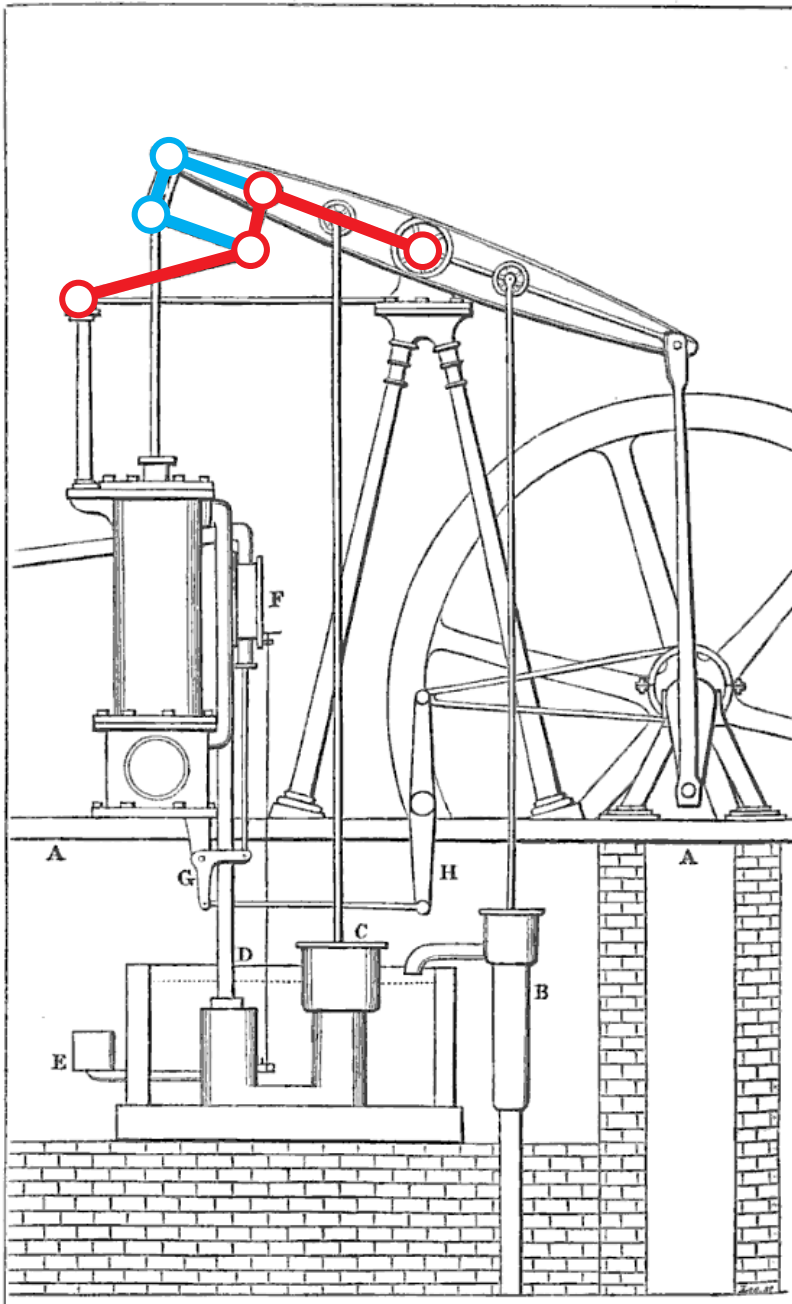


Dimensions of Watt's linkage

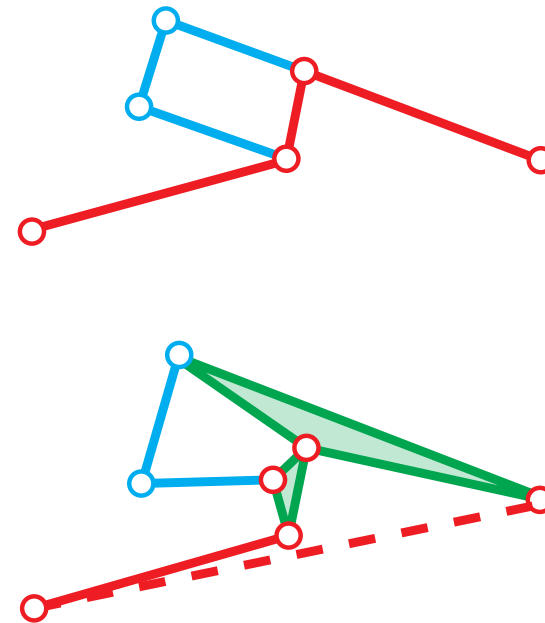


Use in high performance rear suspensions

# The Watt Six-bar Linkage



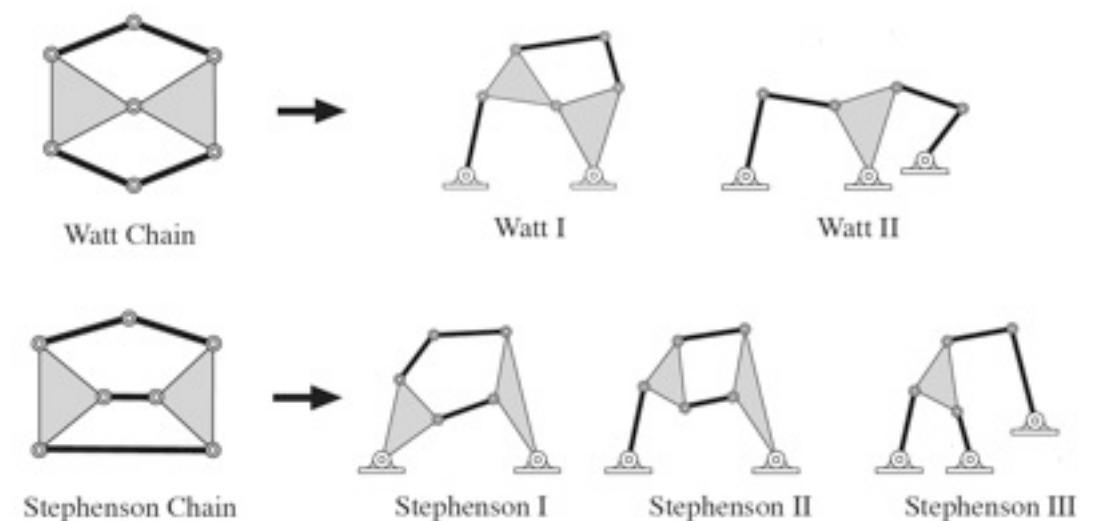
Watt's parallel motion linkage



Watt's parallel motion linkage

The six-bar linkage with Watt topology

Two six-bar linkage topologies





# Computer Aided Design



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## Synthesis of Path-Generating Mechanisms by Numerical Methods<sup>1</sup>

*Algebraic methods in kinematic synthesis are extended to cases in which the development of iterative numerical procedures are required. Algorithms are developed for the numerical solution of nonlinear, simultaneous, algebraic equations. Convergence is obtained without the need for a "good" initial approximation.*

*The theory is applied to the nine-point path synthesis of geared five-bar motion, in terms of which four-bar motion may be considered as a special case.*

### Introduction

THE approximate synthesis of a given path by use of hinged mechanisms has been studied extensively in connection with four-bar mechanisms. Analytical [1]<sup>2</sup> and graphical [2] solutions have been obtained for the problem specified in terms of five precision points and four crank angles; however, problems specified in terms of nine points (and no angles) have not been previously solved. Two published formulations of the nine-point path-synthesis problem are known to the authors [2, 3]. Both are for the four-bar mechanism; however, in the first no attempt is made to solve the equations, and in the second the suggested method of solution seems incomplete.

In this investigation we consider geared five-bar mechanisms, Fig. 1. Since they can generate a large variety of coupler curves [4, 5, 6], these linkages can be used for the solution of varied and complex design problems [7]. Their analysis is more involved than that of four-bar mechanisms, which can be considered as a special case of the geared five-bar—both mechanisms have equivalent coupler curves when the gear ratio is plus one [8, 9, 10, 11]. Previous geared five-bar path syntheses consist of a graphical-design procedure based on the two-degree-of-freedom property of the five-bar "loop" [12], and two analytical formulations of the prescribed crank-rotation problem [13, 19].

Four-bar linkages have (single) coupler links whose both hinge points describe a circular path. In contrast, five-bar linkages have two "floating" coupler links, where only one of the two hinge points (on each link) describes a circular path. Therefore, if the four-bars are called "double circle point" mechanisms, the geared five-bar mechanisms are "single circle point" mechanisms. The present work can be interpreted as a generalization of Burmester's theory to the (single circle point) geared five-bar group: By deriving and solving the equations of synthesis we obtain the center and circle points for the coupled motion of two planes, in this case determined by eight path-increment vectors and no crank angles.

The geared five-bar problem possesses a degree of complexity which renders closed-form algebraic solutions unattainable. Solutions closed-form have been developed for path-angle syntheses with the aid of complex-number and matrix concepts [1].

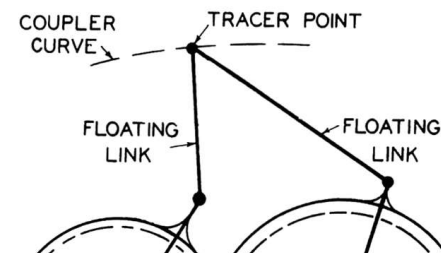
parameters are eliminated at the start and the closure equations reduced to one (nonlinear) equation per precision condition [3]. Secondly, mathematical methods were developed in order to obtain convergence of the numerical iterations used in solving these equations. These mathematical methods, which are included in a digital computer program, contain the following new features:

- 1 The "bootstrap" procedure—this essentially eliminates the need for a "good" initial approximation.
- 2 The "position interchange" procedure—this reformulates the problem in order to eliminate the cause of nonconvergence.
- 3 The "quality-index-control" procedure—this assures convergence to solutions characterized by a reasonable ratio of maximum to minimum link length.

### The Theory of Path Synthesis

**Definition.** Dimensional kinematic synthesis is the procedure of determining the dimensions of a mechanism from the required motion. When the synthesis is phrased in terms of generating a given curve, the procedure is called path synthesis.

Usually one does not attempt to generate the given curve exactly. In fact, only a limited class of motions could be so generated [18, 22], and in general it suffices if within a desired interval the generated curve is a good approximation to the given one. In this paper the approximate path-synthesis problem is formulated by specifying the location of the *precision points* (points at which the given curve and the generated path coincide);



Roger Kaufman using interactive computer graphics for linkage synthesis at MIT in 1970.

F. Freudenstein and G. N. Sandor, "Synthesis of Path Generating Mechanisms by Means of a Programmed Digital Computer," ASME Journal of Engineering for Industry, 81:159-168, 1959.

B. Roth and F. Freudenstein, "Synthesis of Path-Generating Mechanisms by Numerical Methods," ASME Journal of Engineering for Industry, 85:298-304, 1963.

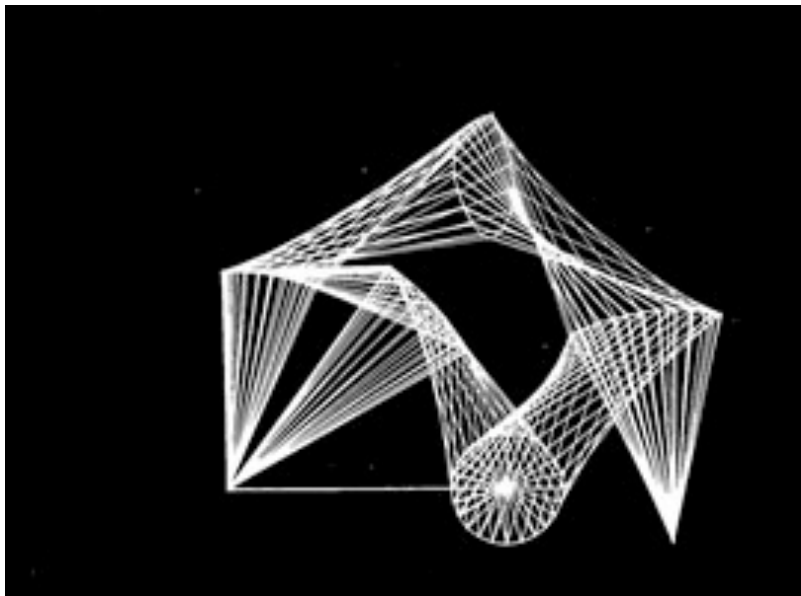
**IBM 7090 digital computer: \$3m, 36bit, 32k core memory**

R. E. Kaufman and W. G. Maurer, "Interactive Linkage Synthesis on a Small Computer", ACM National Conference, Aug.3-5, 1971

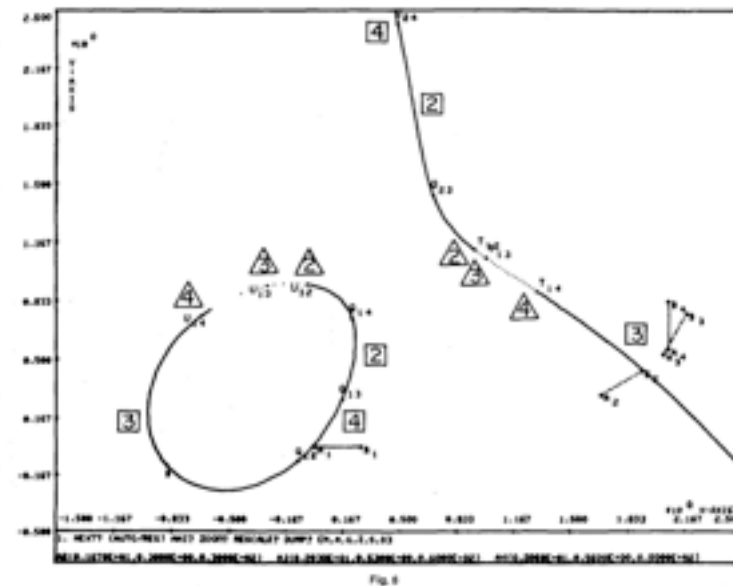
**IBM 1130 digital computer: \$32k, 16bit, 8k core memory**



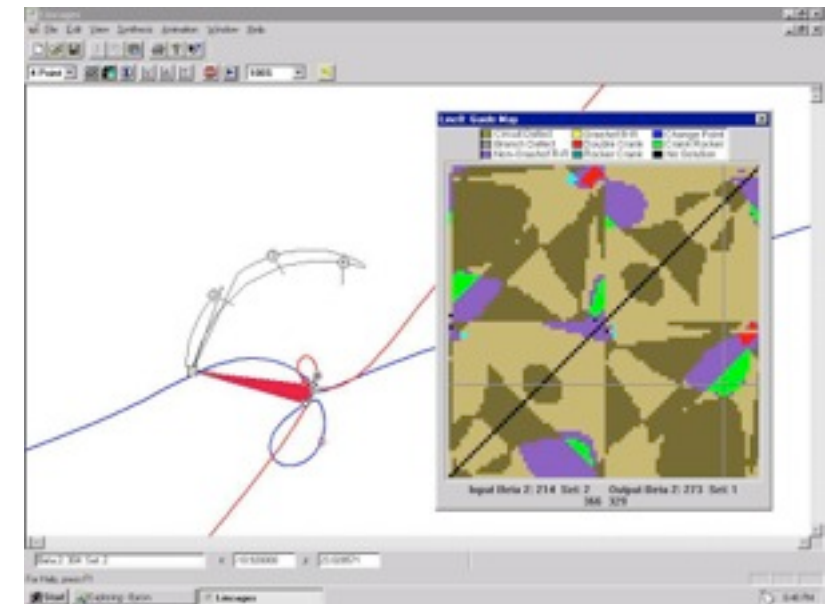
# Linkage Synthesis Software



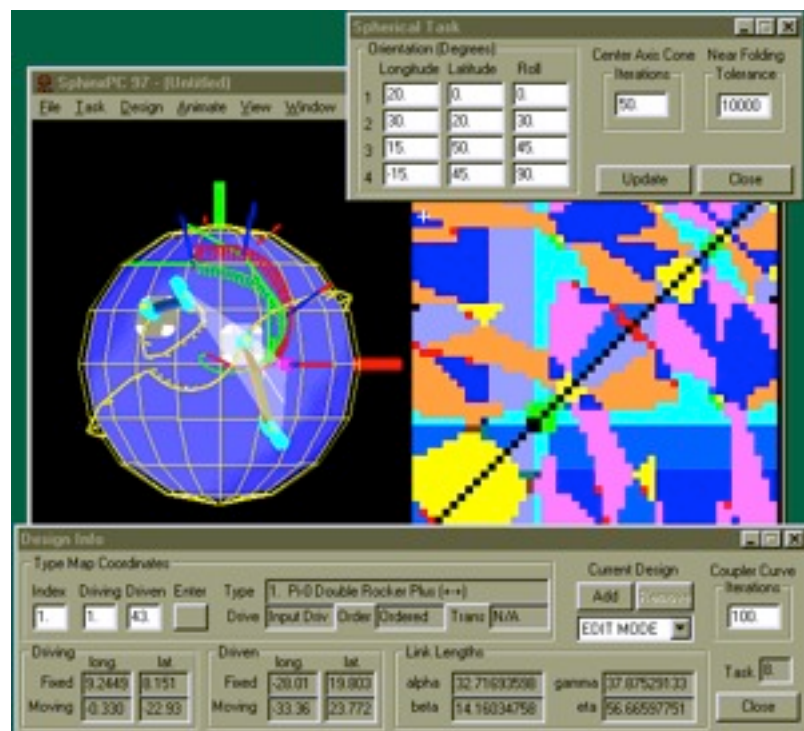
Kinsyn graphical output 1971



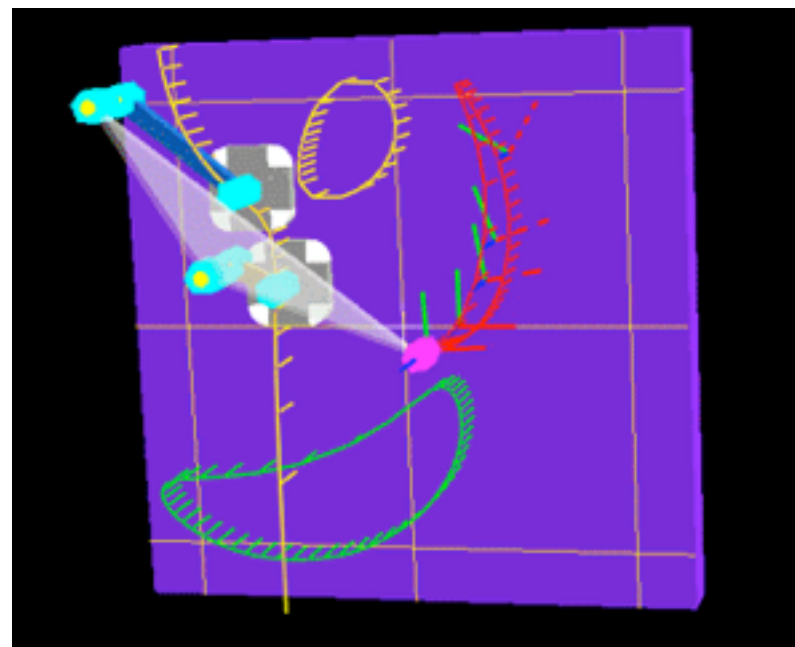
Recsyn graphical output 1981



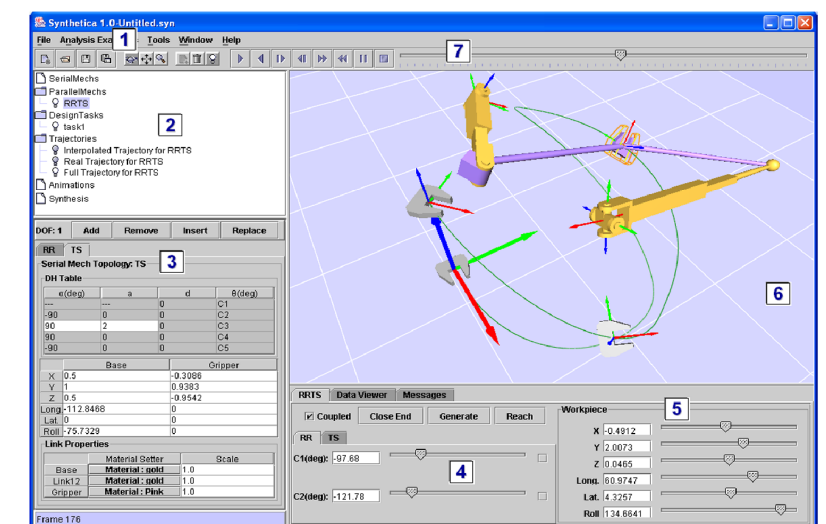
Lincages-4 graphical output 1985



SphinxPC: Spherical 4R design 1997



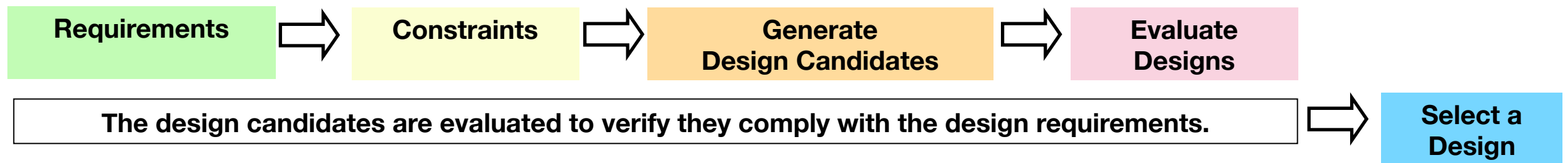
SphinxPC: Planar 4R design 1997



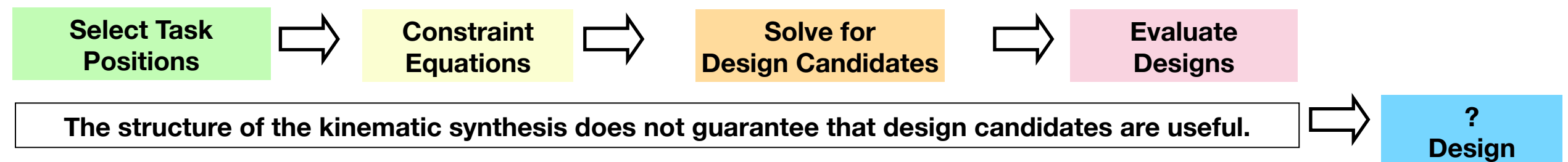
1. Toolbar: file I/O, add/remove/turn on/off objects
2. Object Tree: select object for manipulation
3. Info Panel: show object data (mechanism dimensions, link properties etc.)
4. Teach Panel: drive joint parameters
5. Workpiece: shows the position and orientation of the end-effector
6. Interactive Viewer: dynamically display the objects
7. Animation Bar: animate the selected animation object

Synthetica: Spatial linkage synthesis 2005

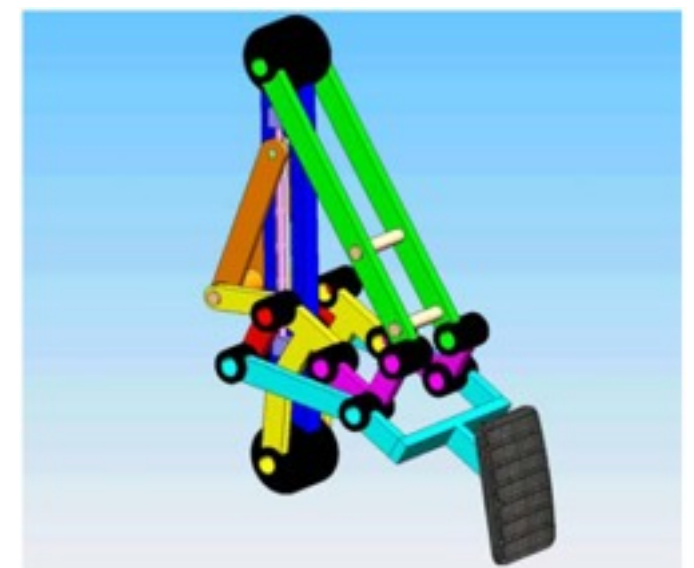
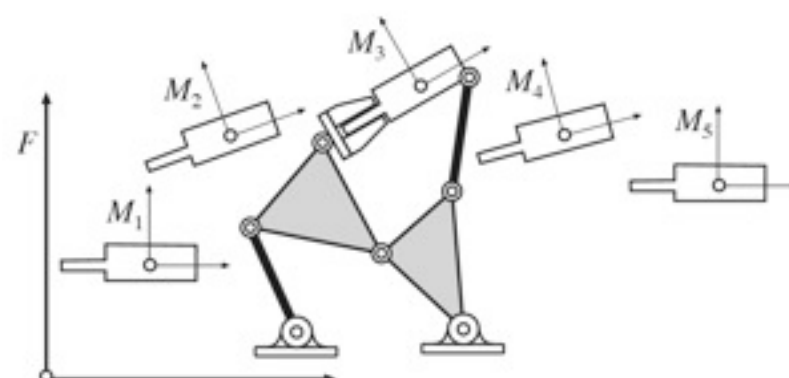
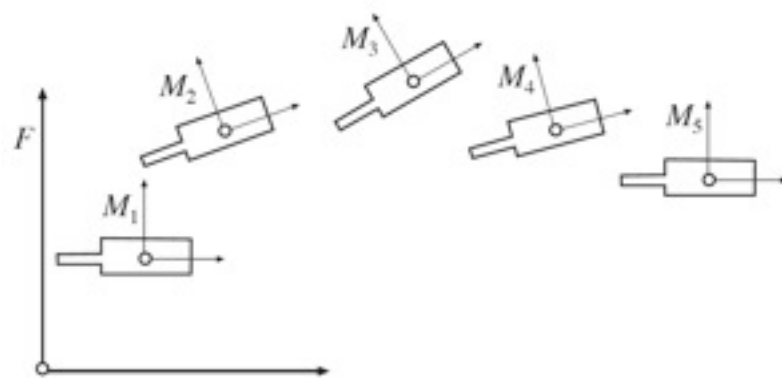
# The Design (Invention) Process



## Kinematic Synthesis Process



$$([D_{1j}]W^1 - G^1) \cdot ([D_{1j}]W^1 - G^1) = R^2, \quad j = 1, \dots, 5.$$



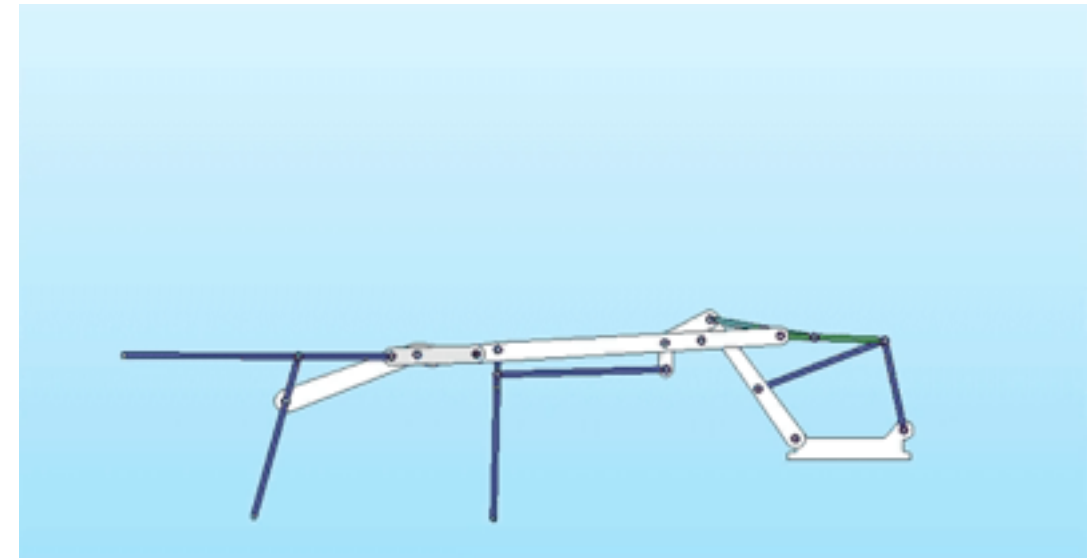


# Design of Articulated Systems

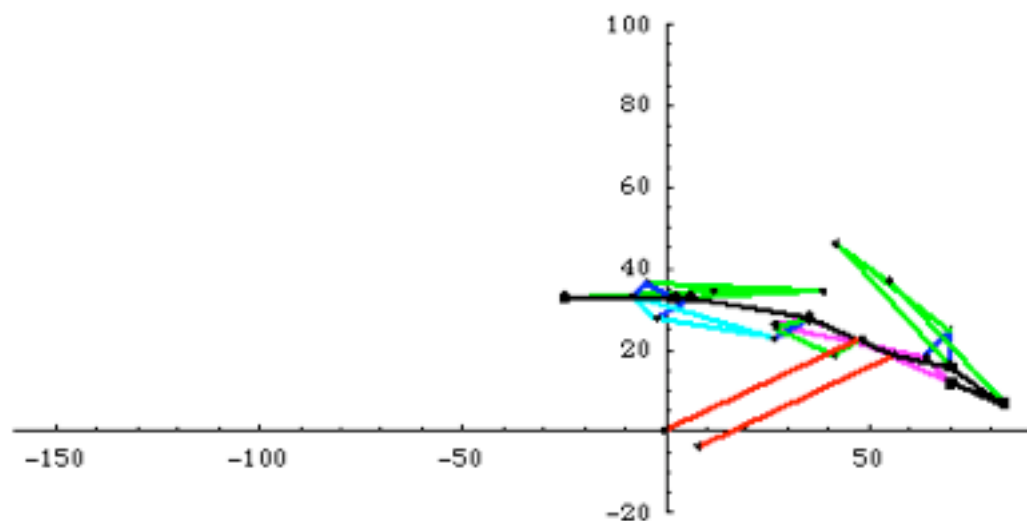
Linkage systems that deploy components such as convertible tops and convertible sofa beds can be seen to have a backbone serial chain, with controlling RR dyads. The result is a mechanically constrained serial chain that can be seen complex.



Internet video of the Volvo C70 convertible



SolidWorks animation of the standard convertible bed linkage



Mathematica animation of the Bruder patent for the C70 linkage



SolidWorks animation of the Mustang convertible top linkage

# A Competition for Emerging Technology Vehicles

- The Energy Invitational is a competition based on power management and efficiency.
  - **Winner:** Maximum speed on a heterogenous course per dollar of energy.
- Venue for mixed competition: Schools, Enthusiasts and Professionals.
- Venue for preparation for other events.
- Open body and chassis designs.
- Open power-plant designs.





# An Energy Endurance Race



## Energy Invitational Goals:

- Integrate student projects ranging from High School to Graduate School.
- Help students develop skills in design, manufacturing, research and testing, team and cost management.
- Provide direct professional experience.

## 2. Linkage Synthesis Theory



# Synthesis Theory Literature

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Kinematic synthesis of serial chains:

- Chen and Roth (1967), Tsai and Roth (1973), Lee and Mavroidis (2000)

Clifford algebras, dual quaternions, and product of exponentials

- Bottema and Roth (1979), Daniilidis (1999), Murray, Li and Sastry (1994)

Synthesis of constrained serial chains

- Su, McCarthy and Watson (2004), Perez and McCarthy (2004)

Planar coupled serial chain mechanisms

- Krovi, Ananthasuresh and Kumar (2002)

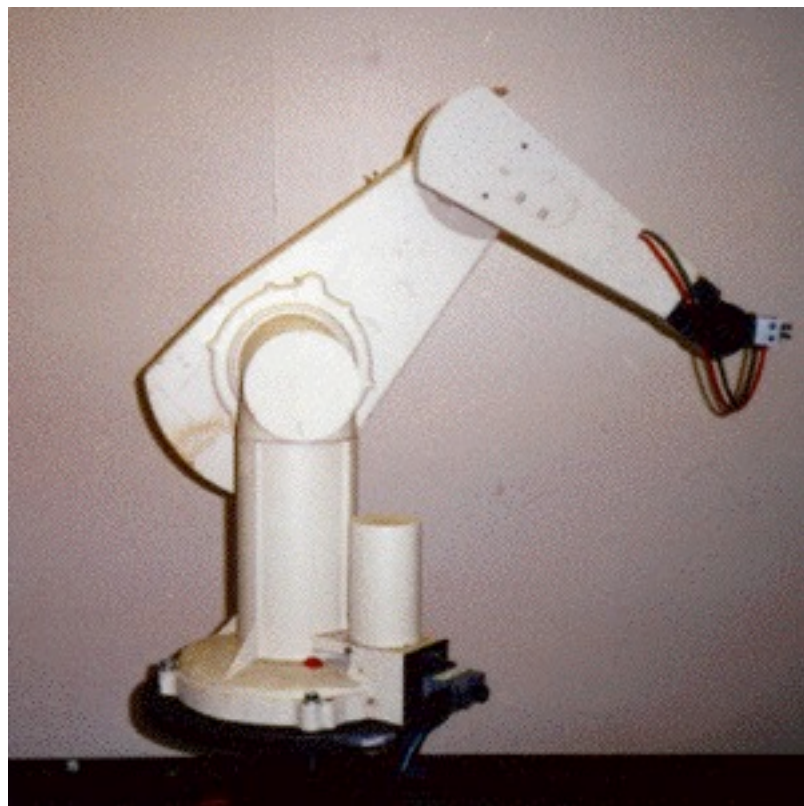
Solving planar synthesis problems using Clifford algebras

- Perez and McCarthy (2005)

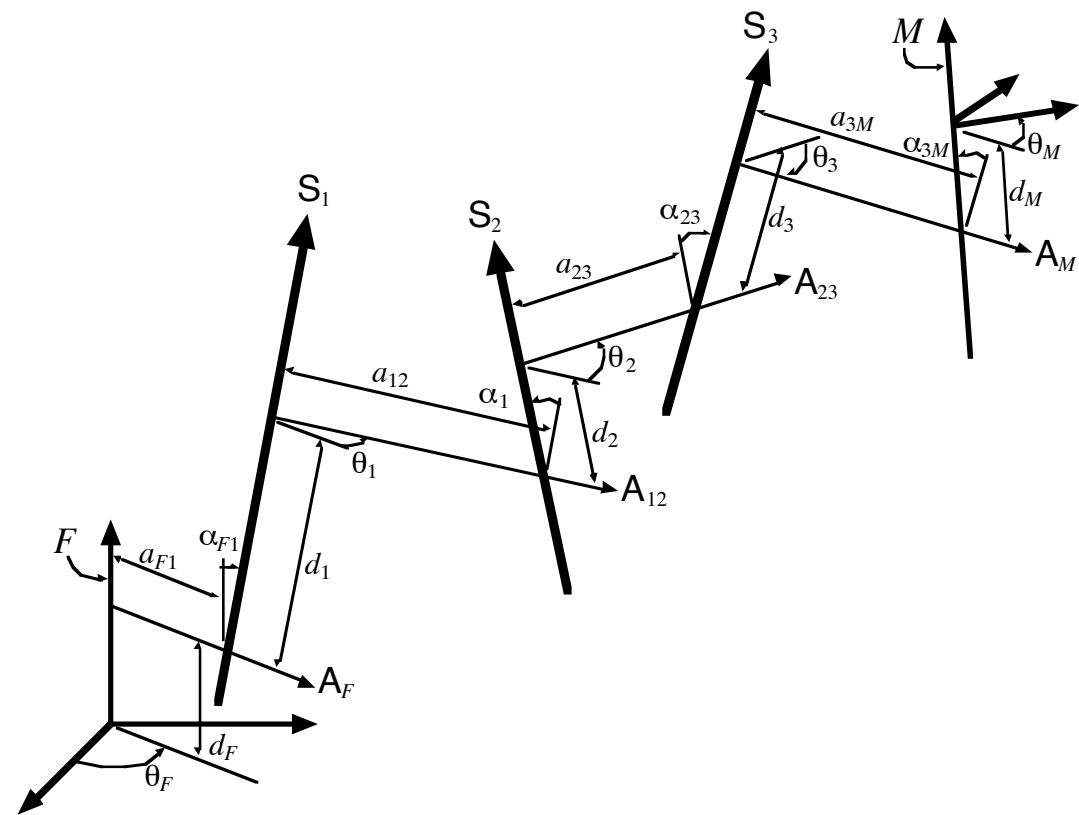


# Open Serial Chain

- The sequence and type of joints, or joint topology, of the serial determines the generic properties of its configuration manifold, such as its degree as an algebraic variety.



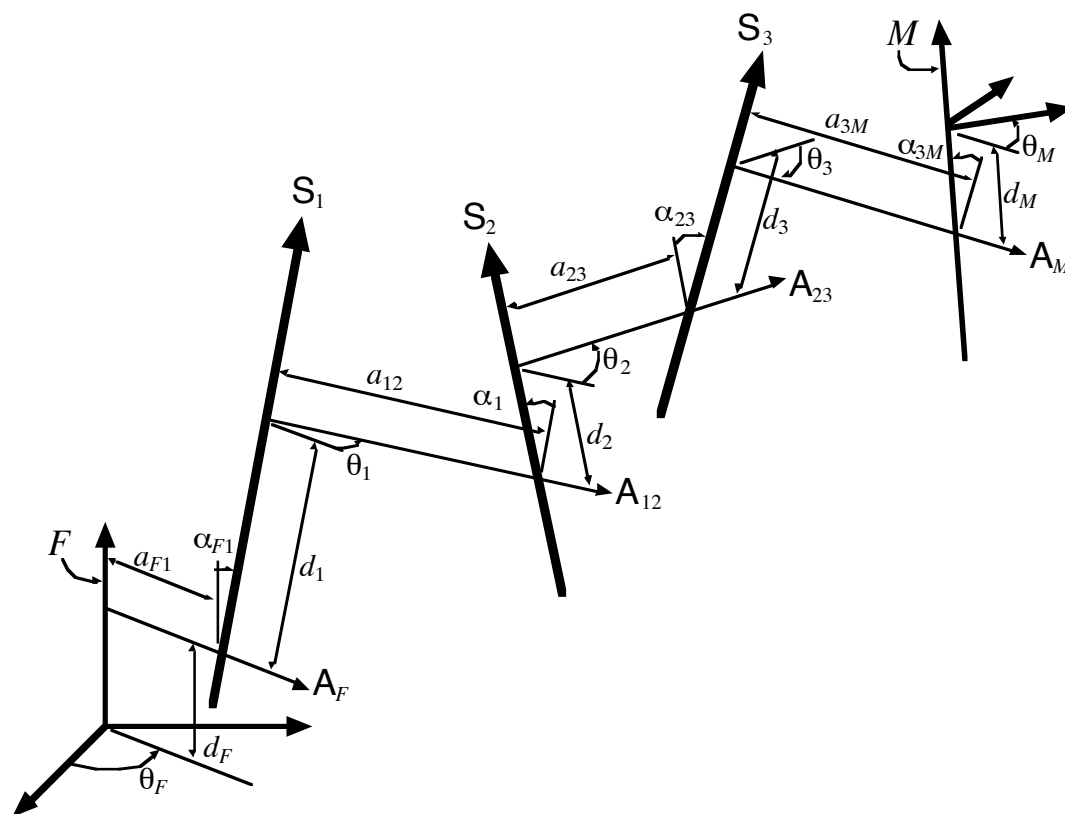
Serial Robot PUMA 560





# The Kinematics Equations

- The dimensions of the serial define the physical properties of its configuration manifold, in particular, the shape and size of its reachable and dextrous workspaces and the local manipulability properties.



$$[Z(\theta_i, d_i)] = \begin{bmatrix} \cos \theta_i & -\sin \theta_i & 0 & 0 \\ \sin \theta_i & \cos \theta_i & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$[X(\alpha_{ij}, a_{ij})] = \begin{bmatrix} 1 & 0 & 0 & a_{ij} \\ 0 & \cos \alpha_{ij} & -\sin \alpha_{ij} & 0 \\ 0 & \sin \alpha_{ij} & \cos \alpha_{ij} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$[D(\vec{\theta})] = [G][Z(\theta_1, d_1)][X(\alpha_{12}, a_{12})][Z(\theta_2, d_2)][X(\alpha_{23}, a_{23})][Z(\theta_3, d_3)][H]$$

These kinematics equations define a parameterized set of positions and orientations called the **configuration manifold**, or workspace, of the robot.

# Kinematic Synthesis Equations

The **Synthesis Problem** specifies (i) a joint topology, (ii) a set of desired properties for the workspace, and (iii) then computes the dimensions of candidate robot designs.

- The topology of the robot yields its kinematics equations:

$$[D(\vec{\theta})] = [G][Z(\theta_1, d_1)][X(\alpha_{12}, a_{12})][Z(\theta_2, d_2)][X(\alpha_{23}, a_{23})][Z(\theta_3, d_3)][H]$$

- The desired properties of the workspace may be presented as a set of positions and orientations,  $[T_k]$ ,  $k=1, \dots, m$ , or as specified angular and linear velocities, or accelerations. The result is a set of **design equations**:

$$\begin{aligned} [T_1] &= [G][Z(\theta_{11}, d_{11})][X(\alpha_{12}, a_{12})][Z(\theta_{21}, d_{21})][X(\alpha_{23}, a_{23})][Z(\theta_{31}, d_{31})][H], \\ [T_2] &= [G][Z(\theta_{12}, d_{12})][X(\alpha_{12}, a_{12})][Z(\theta_{22}, d_{22})][X(\alpha_{23}, a_{23})][Z(\theta_{32}, d_{32})][H], \\ &\dots \\ [T_m] &= [G][Z(\theta_{1m}, d_{1m})][X(\alpha_{12}, a_{12})][Z(\theta_{2m}, d_{2m})][X(\alpha_{23}, a_{23})][Z(\theta_{3m}, d_{3m})][H], \end{aligned}$$

- Solve these equations** to determine the structural parameters  $\alpha_{ij}$ ,  $a_{ij}$ ,  $[G]$  and  $[H]$ , and if necessary we can compute the joint parameters  $\theta_{j,k}$  and  $d_{j,k}$  as well.

# Clifford Algebra Exponentials

- The Clifford algebra exponential of a screw yields

$$e^{\theta J} = \cos \hat{\theta} + \sin \hat{\theta} S = (1 + \mathbf{t}\epsilon)(\cos \theta + \sin \theta S),$$

where

$$\mathbf{t} = dS + \sin \theta \cos \theta C \times S - \sin^2 \theta (C \times S) \times S. \quad \hat{\theta} = \theta + \epsilon d$$

- The spatial displacement of a screw  $w = (w, p \times w + \mu w)$  in a moving frame  $M$  to the screw  $W = (W, P \times W + \mu W)$  in a fixed frame  $F$  is obtained by the operation

$$W = e^{\theta J} w e^{-\theta J}$$

This is equivalent to a rotation by  $2\theta$  combined with a slide of  $2d$  along the axis  $S$

- Thus, displacements are defined using the half dual angle, and we have the Clifford algebra relative kinematics equations for a spatial serial chain

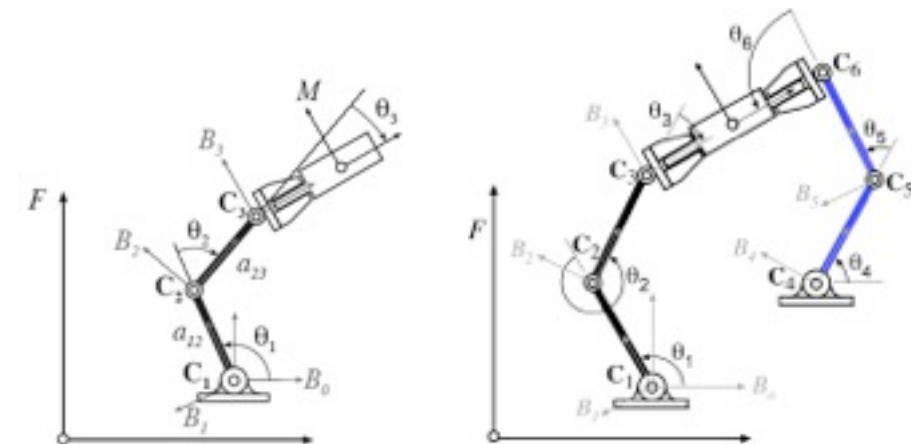
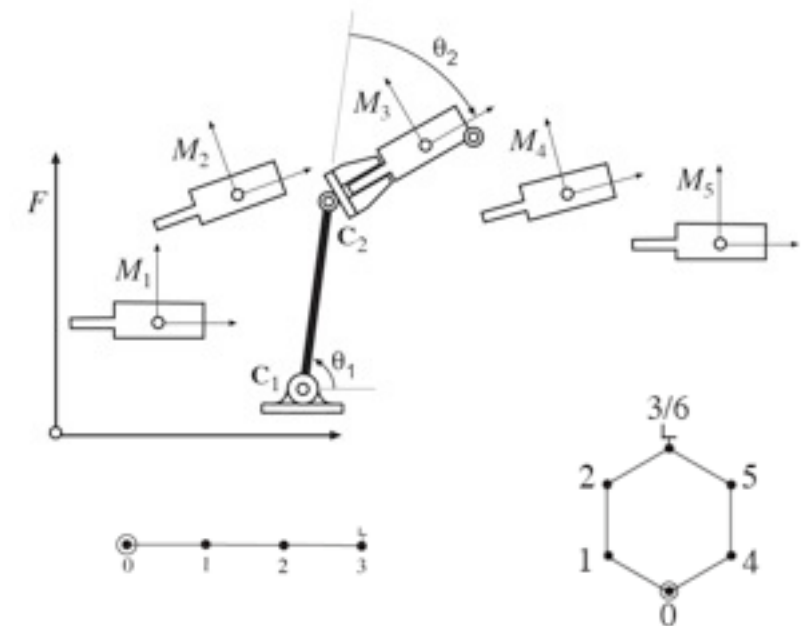
$$\begin{aligned} \hat{D}(\Delta\hat{\theta}) &= e^{\frac{\Delta\hat{\theta}_1}{2} S_1} e^{\frac{\Delta\hat{\theta}_2}{2} S_2} \dots e^{\frac{\Delta\hat{\theta}_n}{2} S_n}, \\ &= (c\frac{\Delta\hat{\theta}_1}{2} + s\frac{\Delta\hat{\theta}_1}{2} S_1)(c\frac{\Delta\hat{\theta}_2}{2} + s\frac{\Delta\hat{\theta}_2}{2} S_2) \dots (c\frac{\Delta\hat{\theta}_n}{2} + s\frac{\Delta\hat{\theta}_n}{2} S_n). \end{aligned}$$

# Synthesis of Planar RR Constraints

- A planar RR chain has five design parameters, therefore five equations are solved to specify these parameters.
- These five equations are obtained by requiring the planar RR chain to reach five specified task positions.

$$([D_{1j}]W^1 - G^1) \cdot ([D_{1j}]W^1 - G^1) = R^2, \quad j = 1, \dots, 5.$$

- The structure of these equations yield four solutions:
  - Use two of solutions yield a **four-bar** linkage
  - Use these equations to design two constraints to an RRR chain to design a **six-bar** linkage
  - Use these equations to design two constraints to a 6R loop to design an **eight-bar** linkage.



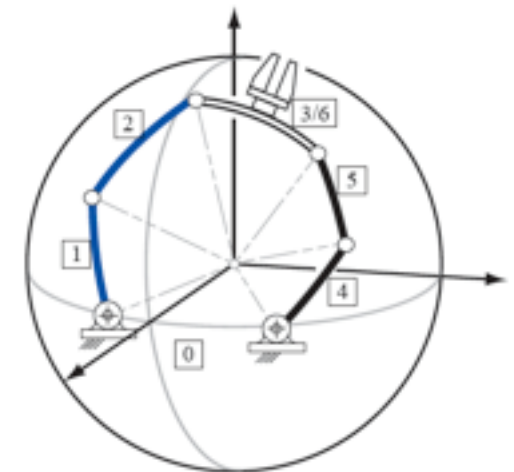
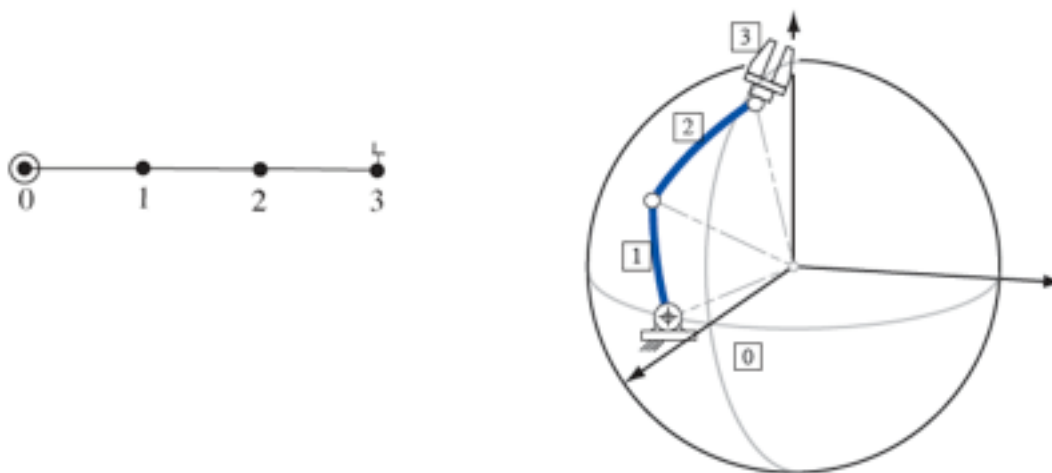
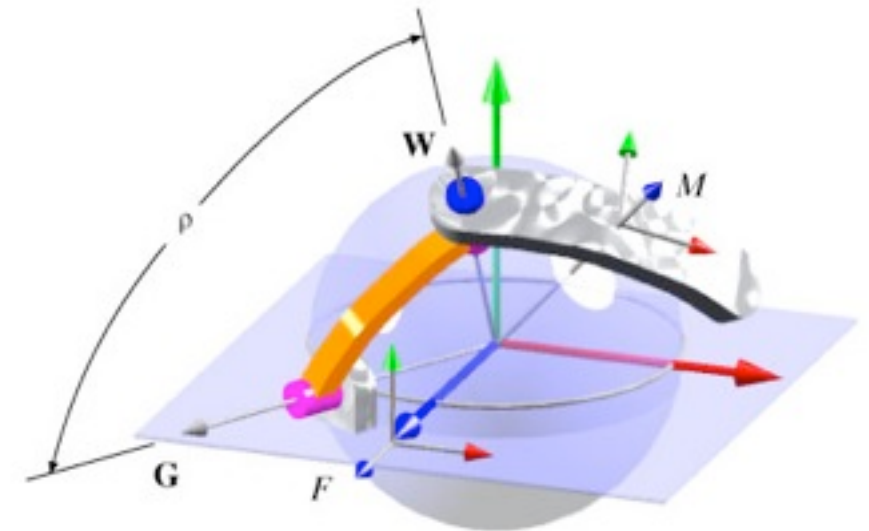
**Our strategy is to design the planar six-bar and eight-bar chain by the addition of two planar RR chains to the designer's planar 3R and 2-RRR backbone respectively.**

# Synthesis of Spherical RR Constraints

- A spherical RR chain has five design parameters, therefore five equations are solved to specify these parameters.
- These five equations are obtained by requiring the planar RR chain to reach five specified task positions.

$$\mathbf{G}^1 \cdot [D_{1j}] \mathbf{W}^1 = \|\mathbf{G}^j\| \|\mathbf{W}^j\| \cos \rho, \quad j = 1, \dots, 5.$$

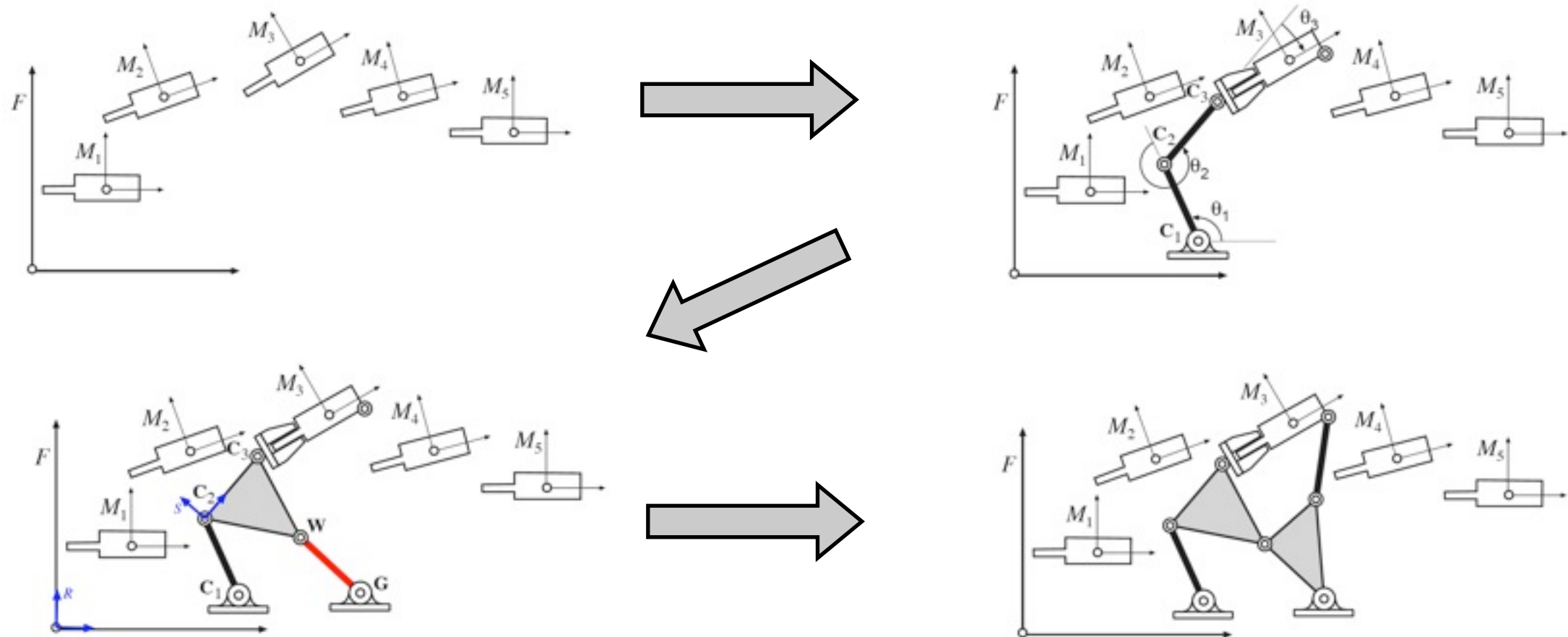
- The structure of these equations yield six solutions:
  - Use two of solutions yield a **four-bar** linkage
  - Use these equations to design two constraints to an RRR chain to design a **six-bar** linkage
  - Use these equations to design two constraints to a 6R loop to design an **eight-bar** linkage.



# Synthesis of Watt-1 6-bar Linkages

## Kinematic Synthesis

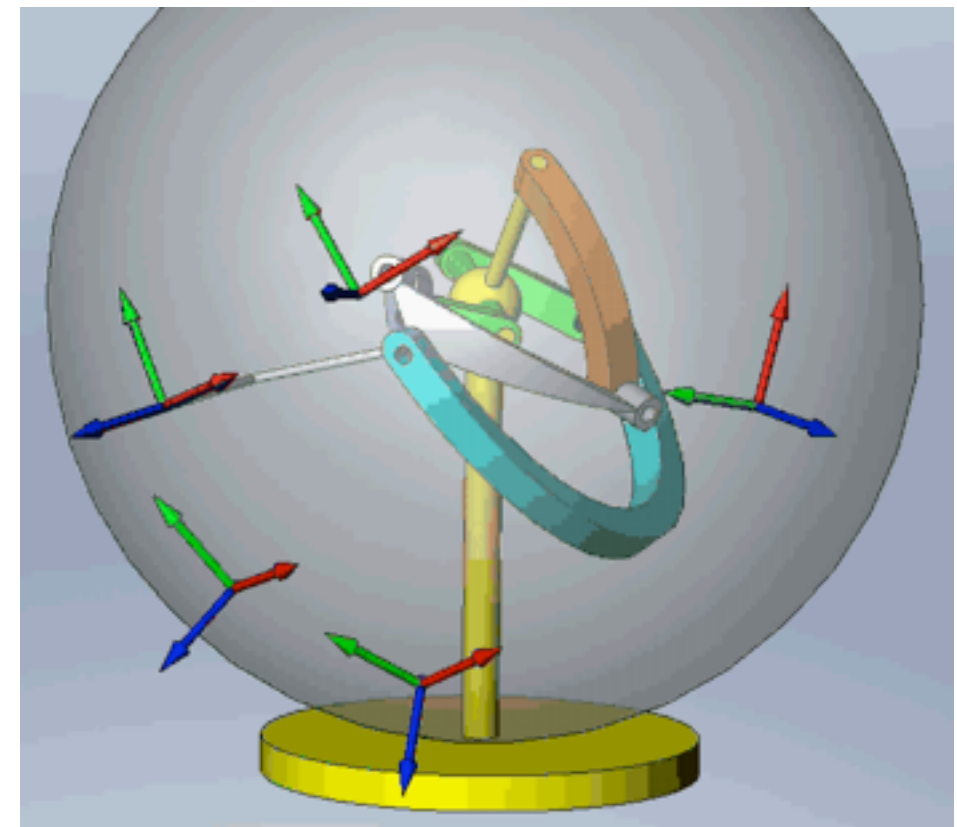
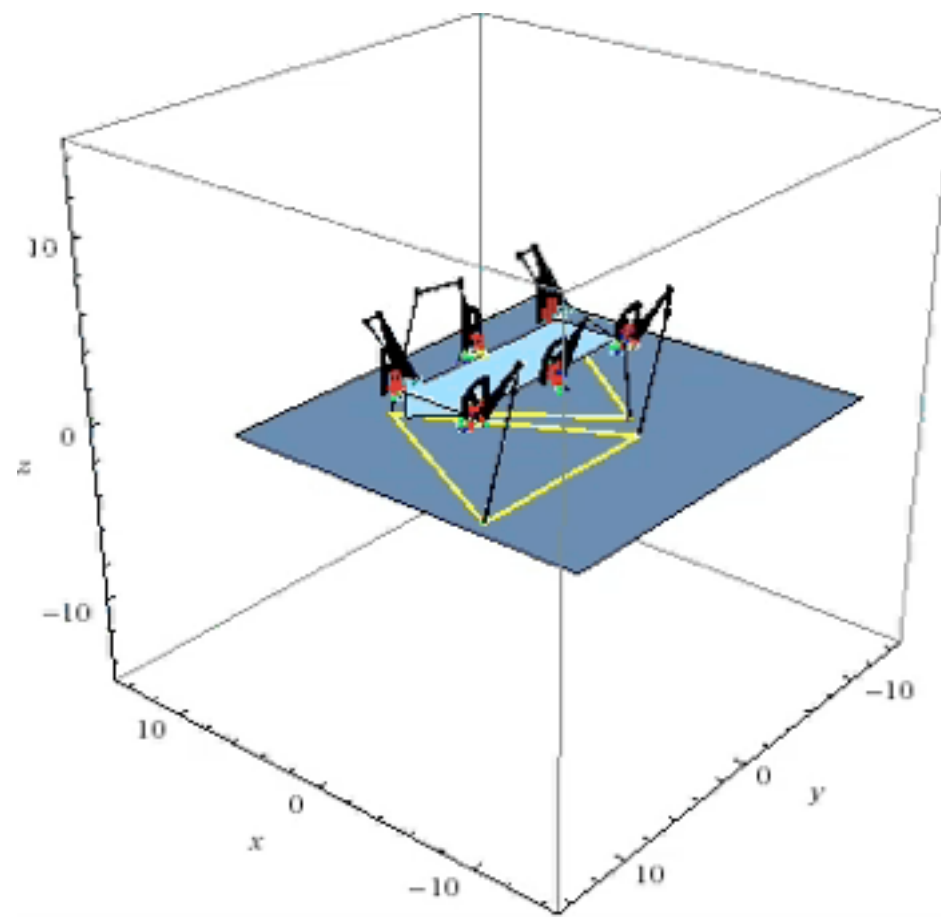
- Specify a set of task positions.
- Select a 3R backbone chain and solve the inverse kinematics problem to determine the position of the links.
- Solve the kinematic synthesis equations to design one RR dyad that constrains the relative positions of two links.
- Solve the kinematic synthesis equations to design a second RR dyad that constrains the relative positions of a second pair of links.





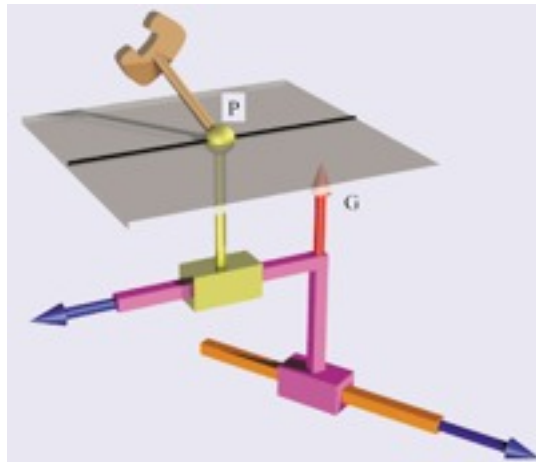
# Spherical Six-bar Linkage Examples

- The design of planar and spherical six-bar chains as constrained planar or spherical 3R robots provides flexibility for the designer.
- Recent practical application is a pedal adjustment mechanism for automotive applications, and a walking machine in the shape of a spider for toy industry.



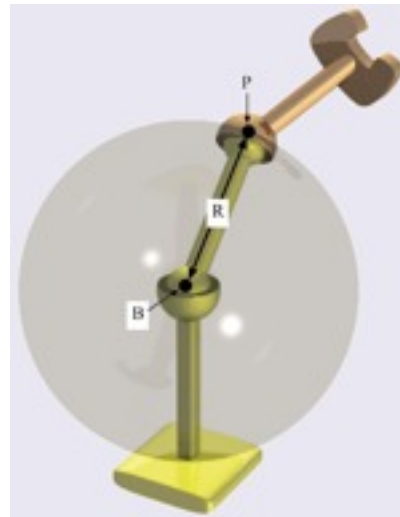
Hip of walking machine in the shape of a spider

# Seven Chains defined by Polynomial Systems

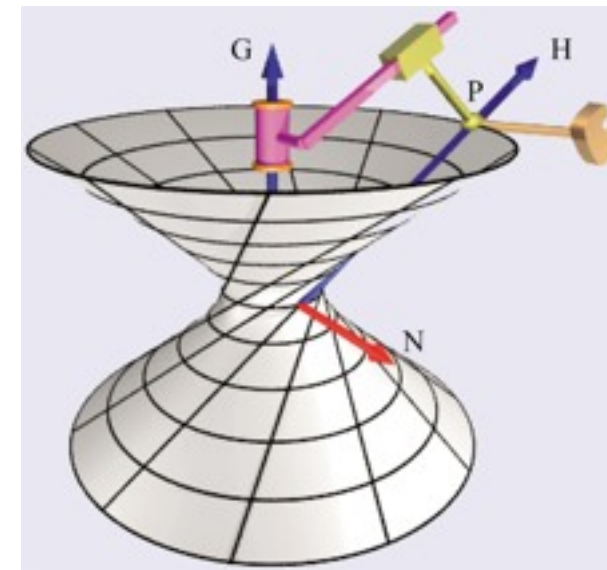


PPS (Plane)

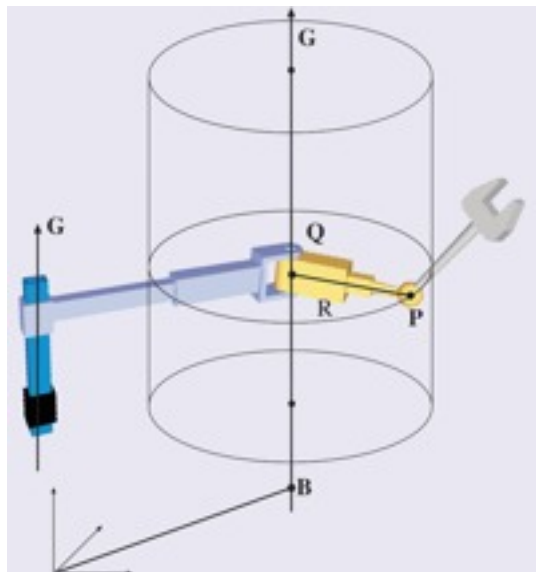
Total degree=32



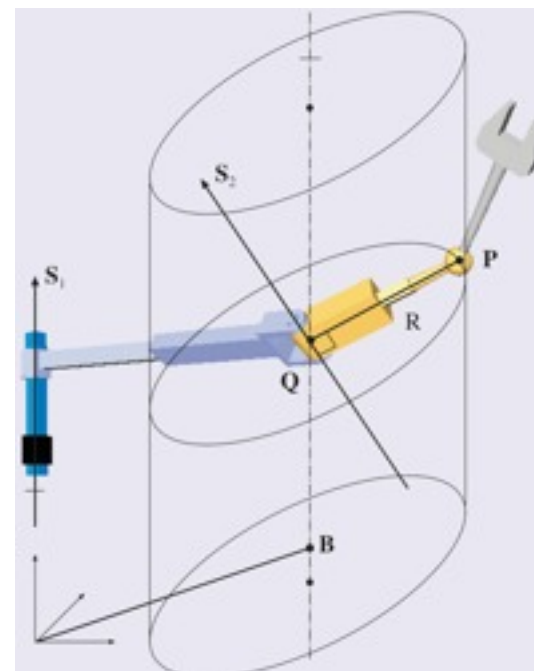
TS (Sphere)



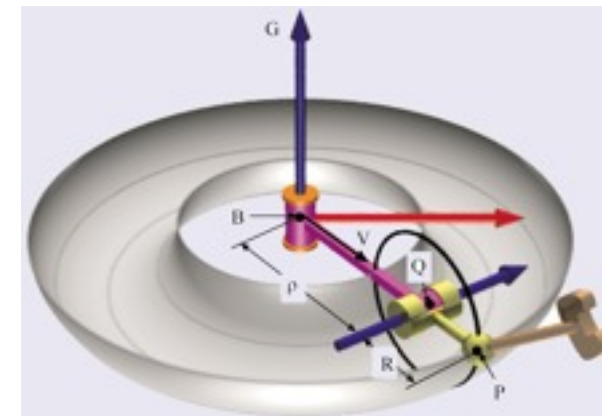
RPS  
(Hyperboloid)



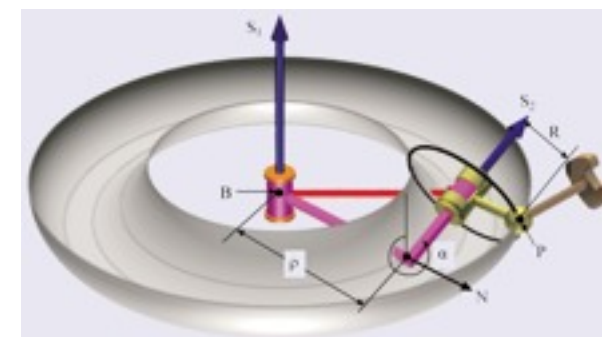
CS  
(Circular Cylinder)



PRS  
(Elliptical Cylinder)



right RRS  
(Right Torus)



RRS  
(Torus)

Total degree=4,194,304

# GLP Bounds and Root Counts

Reachable Surface	n	Total Degree	GLP Bound	Root Count	Computation Cost
PPS (Plane)	6	32	10	10	Resultant
TS (Sphere)	7	64	20	20	Resultant
CS (Circular Cylinder)	8	16,384	2,184	804	POLSYS_GLP 2hrs on PC
RPS (Hyperboloid)	10	262,144	9,216	1,024	POLSYS_GLP 11hrs on PC
PRS (Elliptic Cylinder)	10	2,097,152	247,968	18,202	POLSYS_GLP 33min on BH
right RRS (Circular Torus)	10	1,048,576	868,352	94,622	POLSYS_GLP 72mins on BH
RRS (Torus)	12	4,194,304	448,702	42,786	POLSYS_GLP 42mins on BH

n: maximum number of task position

BH: Blue Horizon system of SDSC(1024 CPU used)

MPC: Beowulf cluster system of UCI medium performance computing

PC: Pentium IV 1.5 GHz

hrs: CPU hours

# Spatial Serial Chains



- Open chains constructed from hinge joints R and sliding joints P have parameterized kinematic synthesis equations.
- The two and three DOF cases have been solved either algebraically or using polynomial homotopy solvers to find all of the roots of the synthesis equations.
- The four and five DOF cases can be solved numerically to find one root. *However, the degree of these equations are known only for special cases.*

Degrees of Freedom	Chain	Struct. Params.	Special Cases
2			
	PP	4	
	PR	6	C
	RR	8	T
3			
	PPP	6	
	PPR	8	CP
	PRR	10	CR, PT
	RRR	12	S, RT
4			
	PPPR	10	CPP
	PPRR	12	CC, CPR, PPT
	PRRR	14	CT, PS, CRR, PRT
	RRRR	16	RS, TT, RRT
5			
	PPPRR	14	CCP, CPPR, PPRT
	PPRRR	16	CCR, CPT, <b>PPS</b> , PPRT, CPRR
	PRRRR	18	<b>CS</b> , CRT, <b>PRS</b> , PTT, PRRT
	RRRRR	20	<b>TS</b> , RTT, <b>RRS</b> , RRRT

### 3. Why Synthesis Theory Fails and How to Fix It

# A Fundamental Challenge: Multiple Solutions

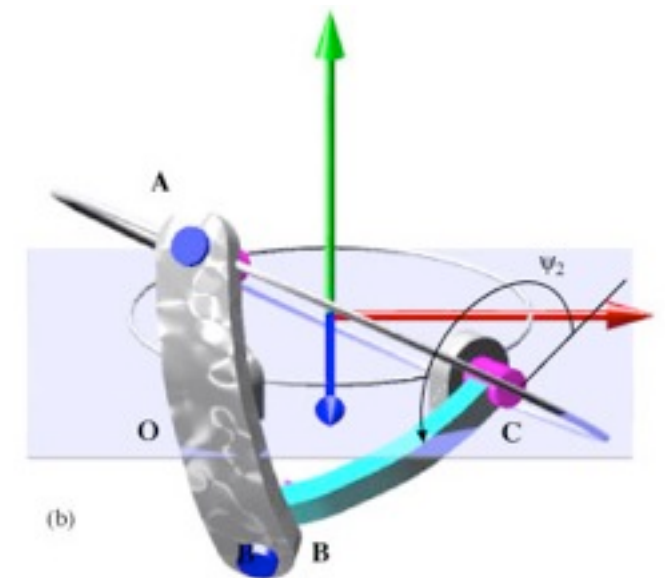
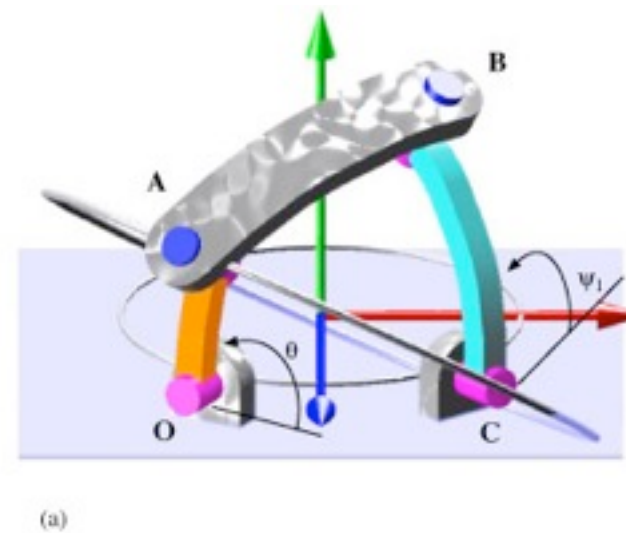
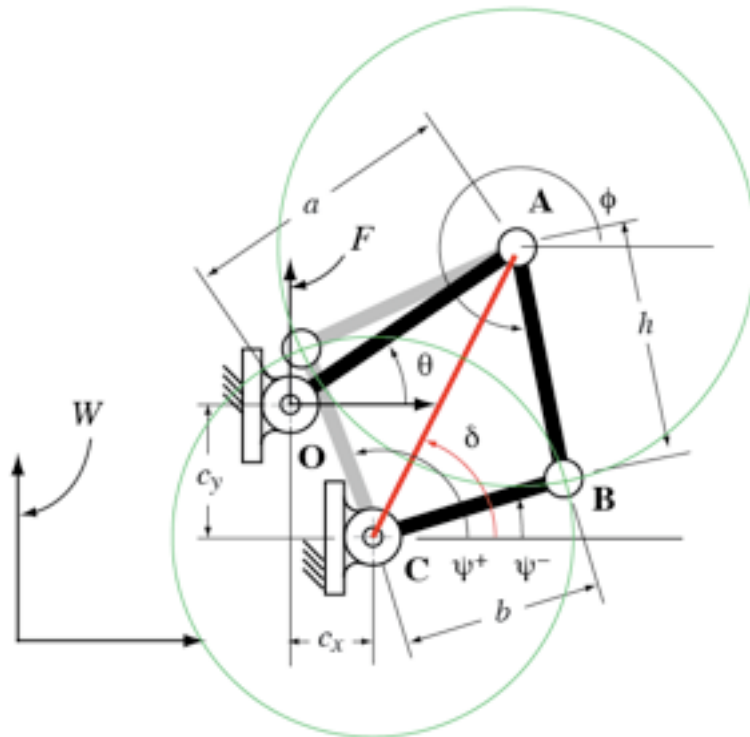
An articulated loop has multiple assemblies:

- The 4R linkage has the constraint equation

$$A(\theta) \cos \psi + B(\theta) \sin \psi = C(\theta).$$

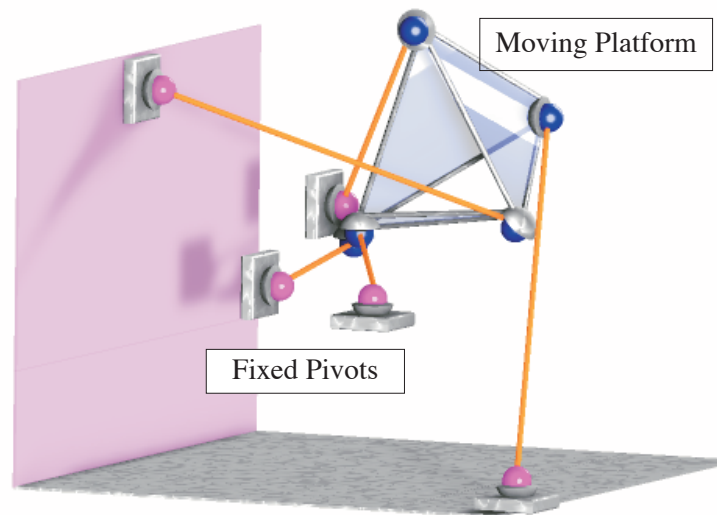
- This is a quadratic equation that yields two solutions

$$\psi = \delta \pm \text{ArcCos}(C/(A^2+B^2)^{1/2}).$$





# The 5-SS Platform



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and Telecommunications,  
Beijing 100088, P.R. China

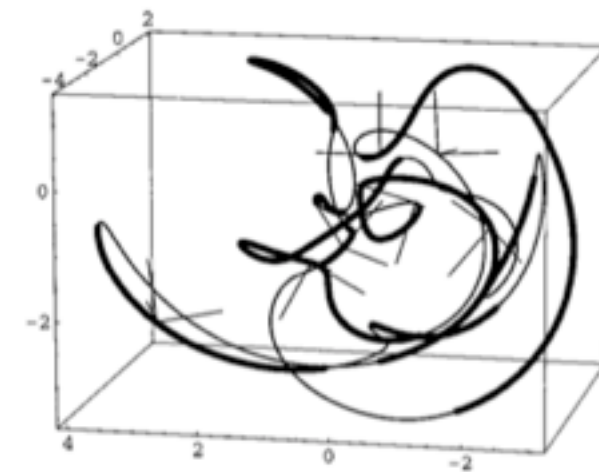
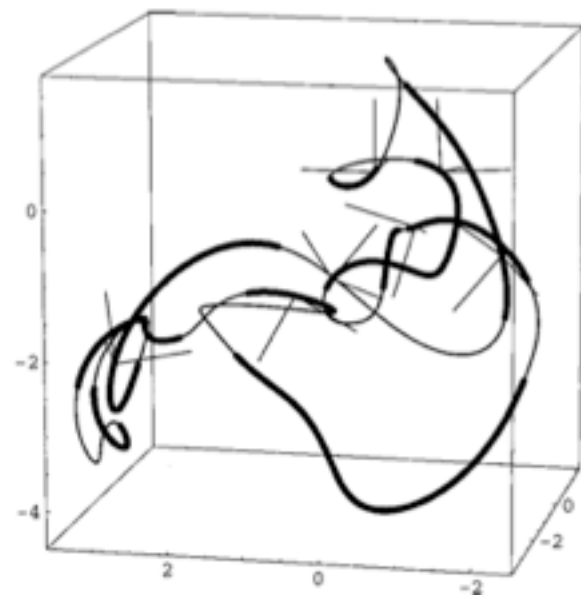
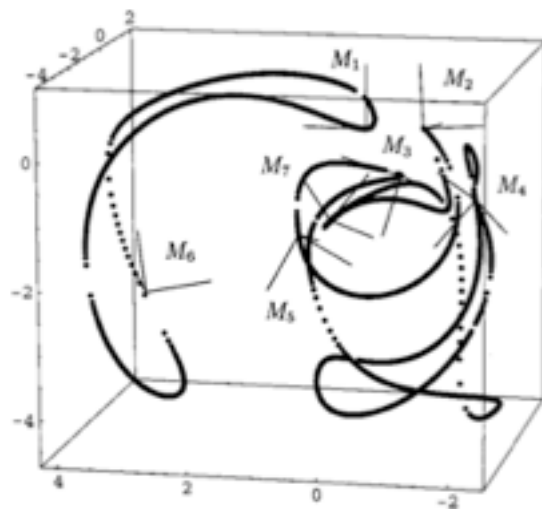
**J. Michael McCarthy**  
Department of Mechanical Engineering,  
University of California, Irvine,  
Irvine, CA 92697

## On the Seven Position Synthesis of a 5-SS Platform Linkage

*This paper builds on Innocenti's polynomial solution for the 5-SS platform that generates a one-degree of freedom movement through seven specified spatial positions of a rigid body. We show that his  $60 \times 60$  resultant can be reduced to one that is  $10 \times 10$ . We then actuate the linkage using a prismatic joint on the sixth leg and determine the trajectory of the reference point through the specified positions. The singularity submanifold of this associated 6-SS platform provides information about the movement characteristics of the 5-SS linkage. [DOI: 10.1115/1.1330269]*

Journal of Mechanical Design

MARCH 2001, Vol. 123 / 75

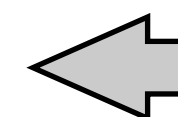


20 real solutions of the synthesis equations yields 15,504 5-SS platform linkages.  
The workpiece must move smoothly through the 7 positions, otherwise the design is useless.

# Planar RR Numerical Experiments

We use Mathematica to generate random tasks and evaluate determine usable 4R designs. We find that the distribution of real and complex roots is slightly biased toward the cases of real plus complex roots.

<i>Random Tasks</i>	<i>10,000</i>	<i>50,000</i>	<i>50,000</i>	<i>100,000</i>	<i>1,000,000</i>
Four complex solutions	2253	11680	11617	23177	230479
% of solutions	23.5	23.4	23.2	23.2	23.0
Two complex, Two real	5486	27449	27481	54957	550944
% of solutions	54.9	54.9	55.0	55.0	55.1
Four real solutions	2261	10871	10902	21866	218577
% of solutions	21.6	21.7	21.8	21.9	21.8
<i>Tasks with usable four-bars</i>	671	3179	3099	6543	63945
Tasks per usable 4R design	14.9	15.7	16.1	15.3	15.6

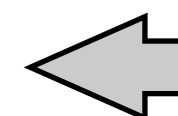


Thus, the probability that a task yields a usable 4R design is found to be  $p = 1:15.5$ .

# Spherical RR Numerical Experiments

We use Mathematica to generate random tasks and evaluate determine usable 4R designs. We find that the distribution of real and complex roots is slightly biased toward the cases of real plus complex roots.

<i>Random Tasks</i>	<i>10,000</i>	<i>50,000</i>	<i>100,000</i>
Six complex solutions	1643	8426	17013
% of solutions	16.4	16.8	17.0
Four complex solutions	6010	29824	59266
% of solutions	60.1	59.6	59.3
Two complex, Two real	5533	10662	21595
% of solutions	21.2	21.3	21.6
Six real solutions	224	1088	2126
% of solutions	2.2	2.2	2.1
<i>Tasks with usable four-bars</i>	302	2080	3182
Tasks per usable task	33.1	31.8	31.4



The probability that a spherical task yields a usable spherical 4R design is found to be  $p = 1:31.5$ .

# New Strategy: Tolerance Zones

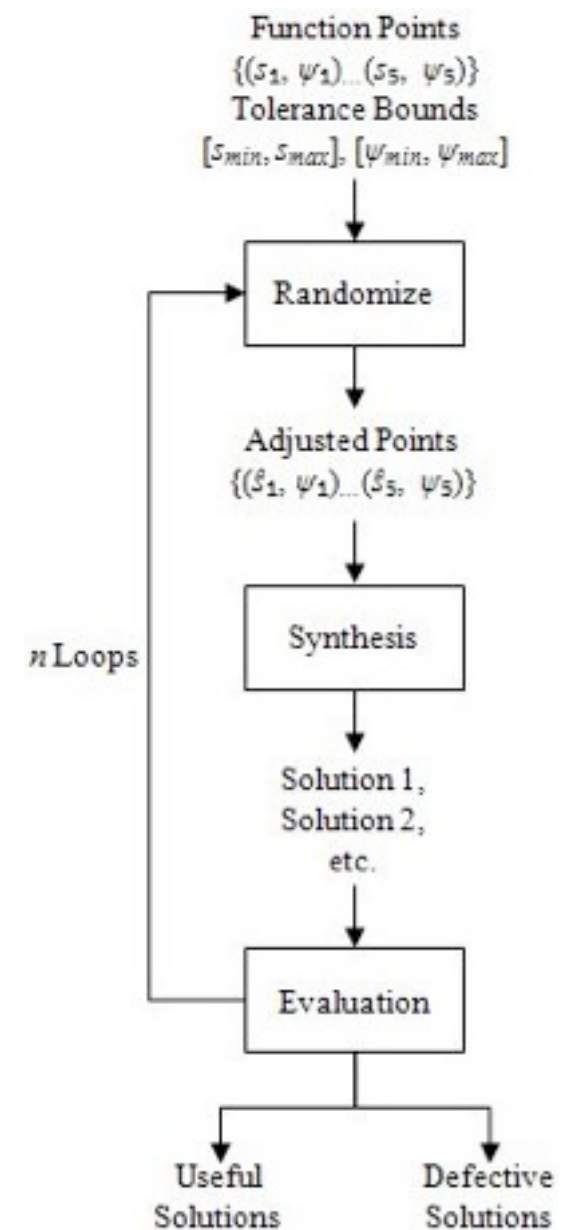
The chances that a user specified task yields a usable linkage are slim, so we search in regions around each task position:

- i. set a tolerance zone around each task position,
- ii. randomly select task positions from the tolerance zones,
- iii. evaluate all candidates to find usable linkages, and
- iv. loop 1000 or more times.

J. E. Holte, T. R. Chase and A. G. Erdman, 2000, "Mixed Exact-Approximate Position Synthesis of Planar Mechanisms, Journal of Mechanical Design, 122:280-286

J. R. Mlinar and A. G. Erdman, 2000, "An Introduction to Burmester Field Theory," Journal of Mechanical Design, 122:25-30.

H. E. Stumph and A. P. Murray, 2000, "Defect-Free Slider-Crank Function Generation for 4.5 Precision Points," Proceedings of the Mechanisms and Robotics Conference, Baltimore, MD, September 10-13, 2000.



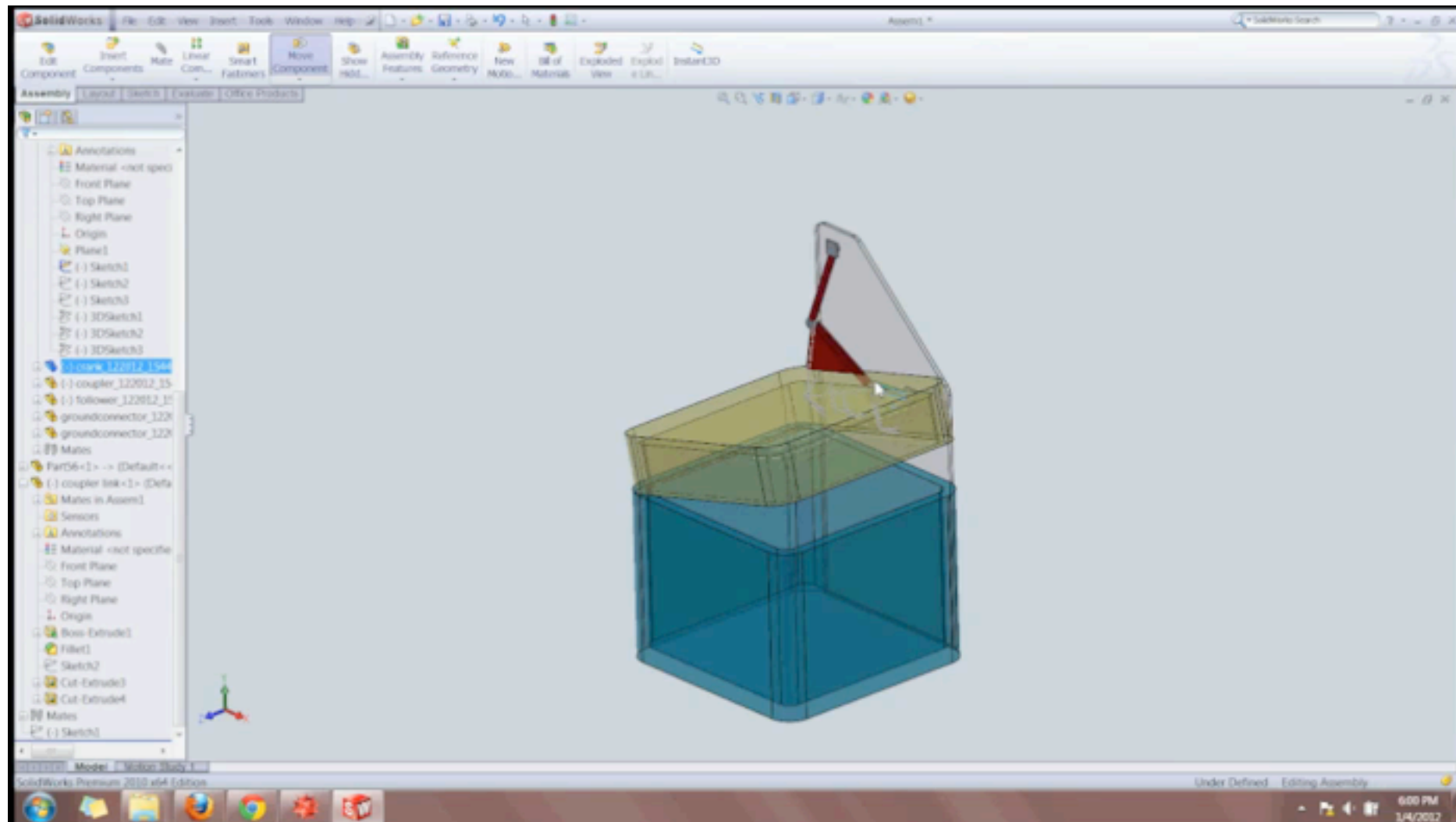
**This quasi-position synthesis for five task positions yields a finite set of linkages for evaluation.**

## 4. Computer Aided Invention



# Planar Four-Bar Linkages

MechGen 2 is a combination of Mathematica, Visual Basic and SolidWorks.



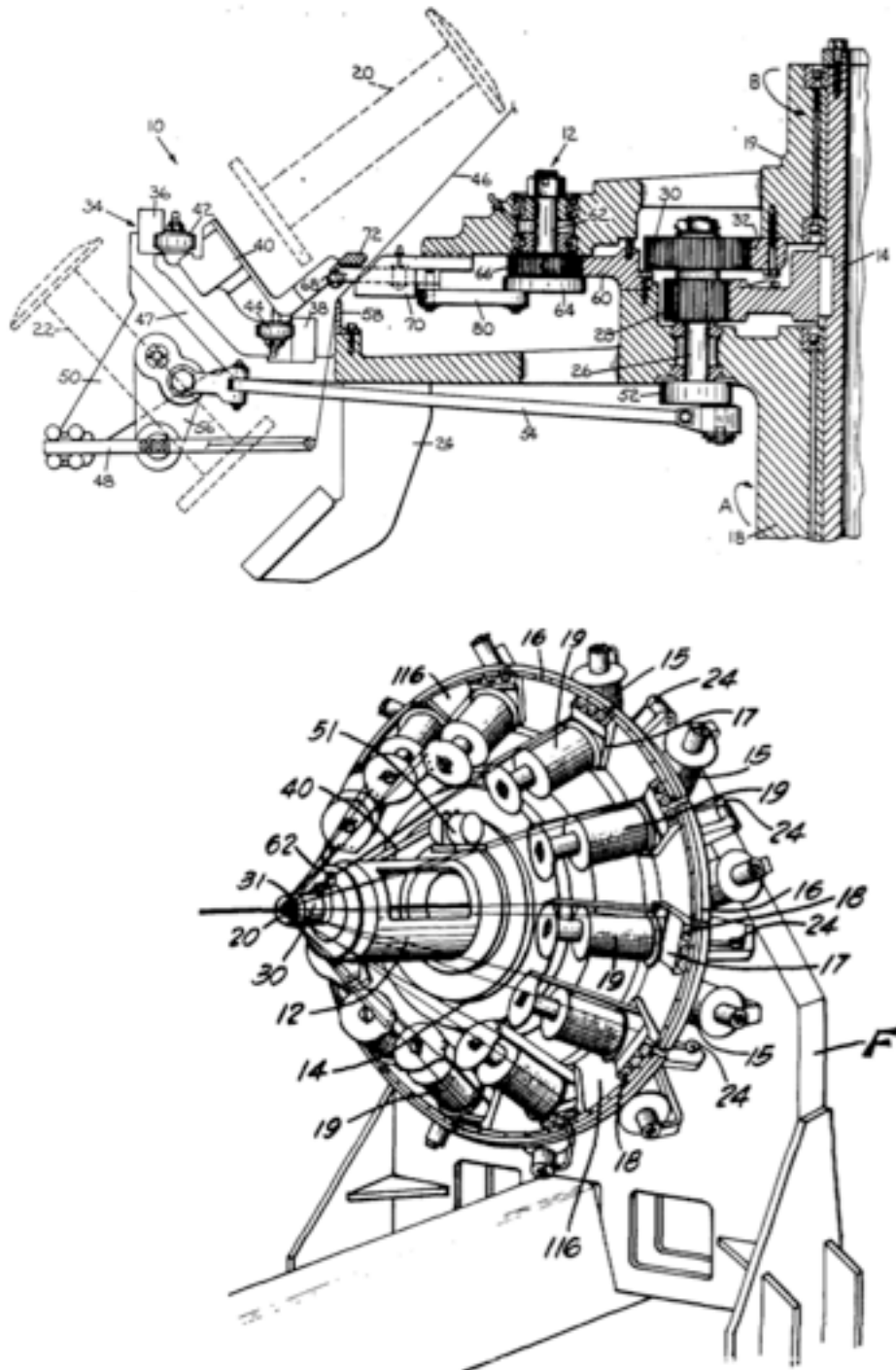
The design process:

1. Draw an the assembly in SolidWorks that has five task positions of a moving body relative to a fixed body;
2. Execute MechGen 2, identify tolerances, and compute usable linkages, then select and generate the assembly of the linkage;
3. Import the linkage into SolidWorks and attach it to the moving and fixed bodies.



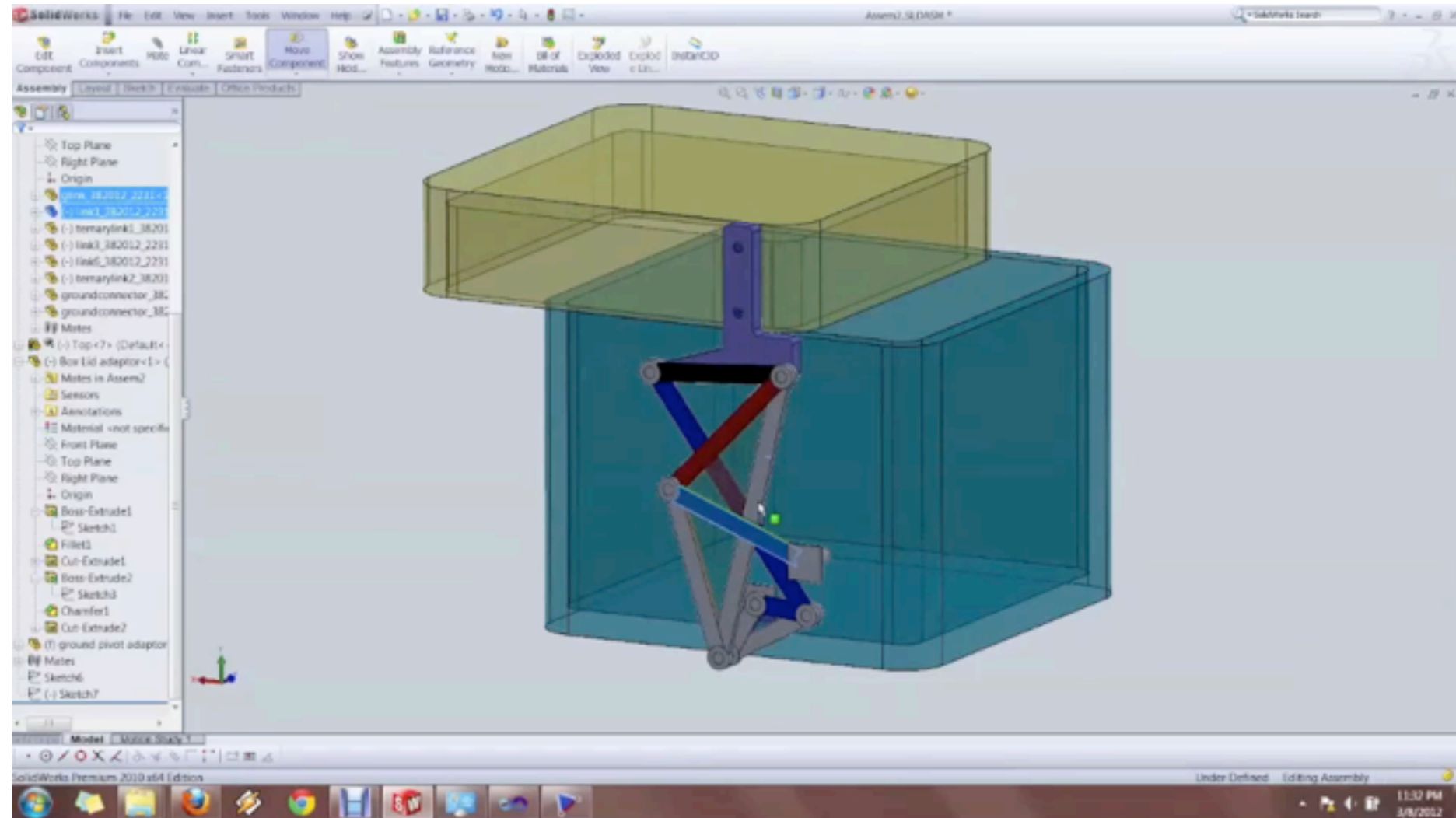
# Slider-crank

Wire guide linkage for a rotary braider for industrial and hydraulic hose.



# Planar Six-Bar Linkages: MechGen 3.0

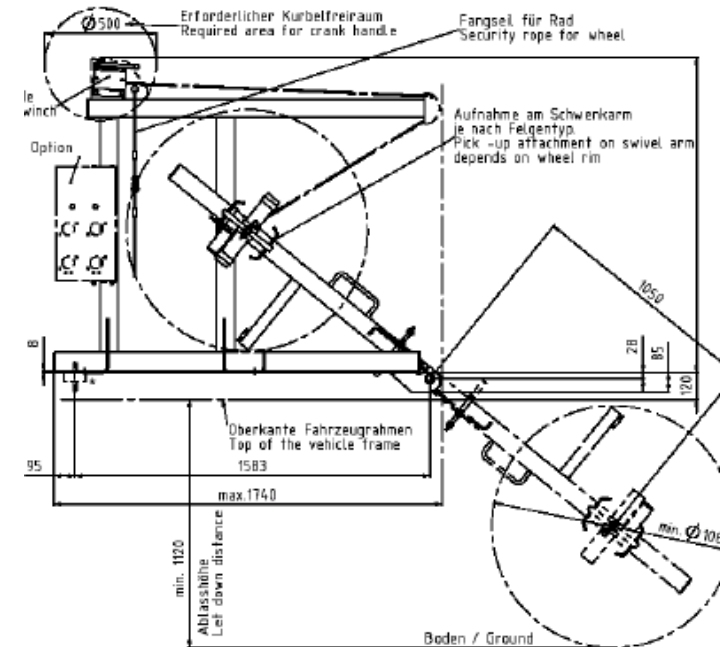
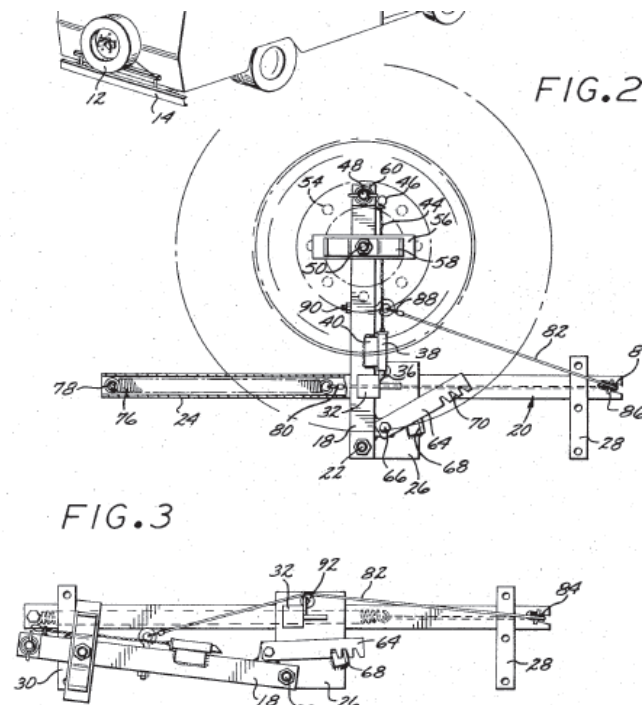
MechGen 3 is also a combination of Mathematica, Visual Basic and SolidWorks.



Specify a five task positions for a 3R chains and generate two RR constraints.

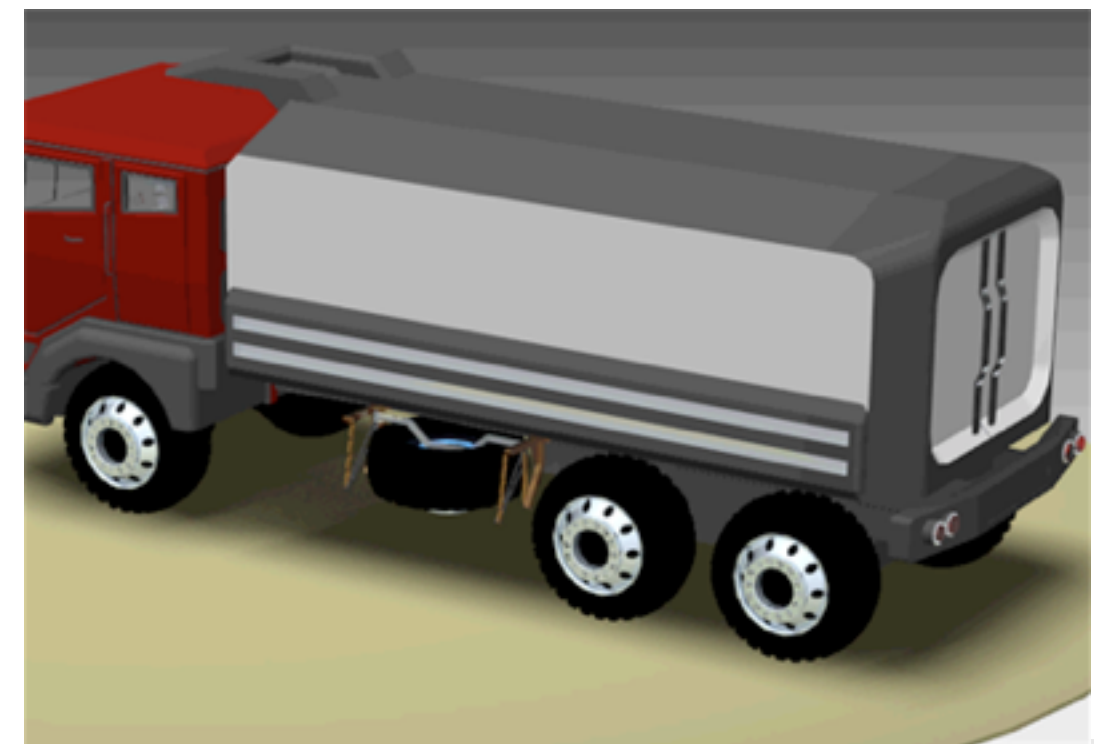
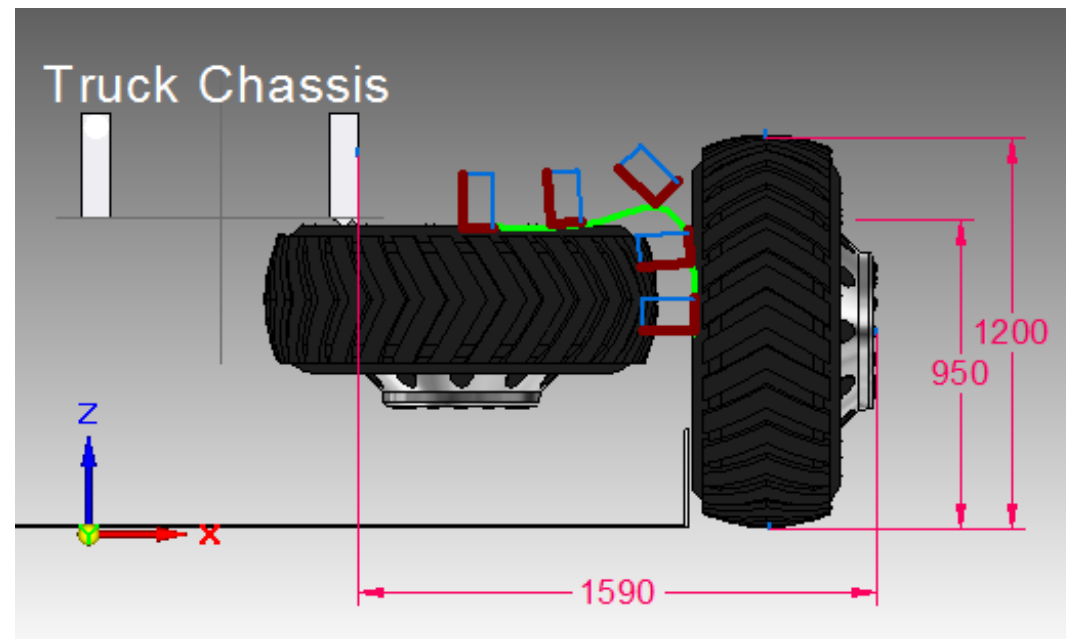
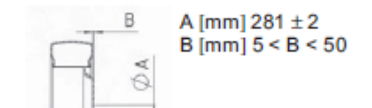


# Truck Spare Tire Deployment



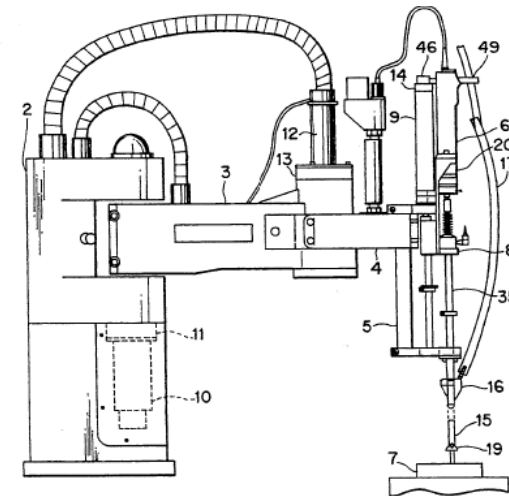
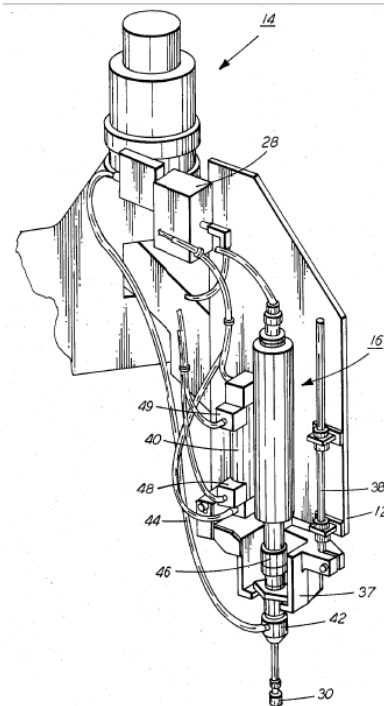
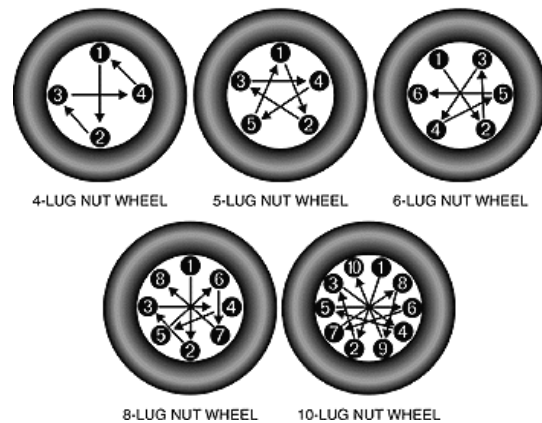
Versions:

Type 1: wheel with disk rim



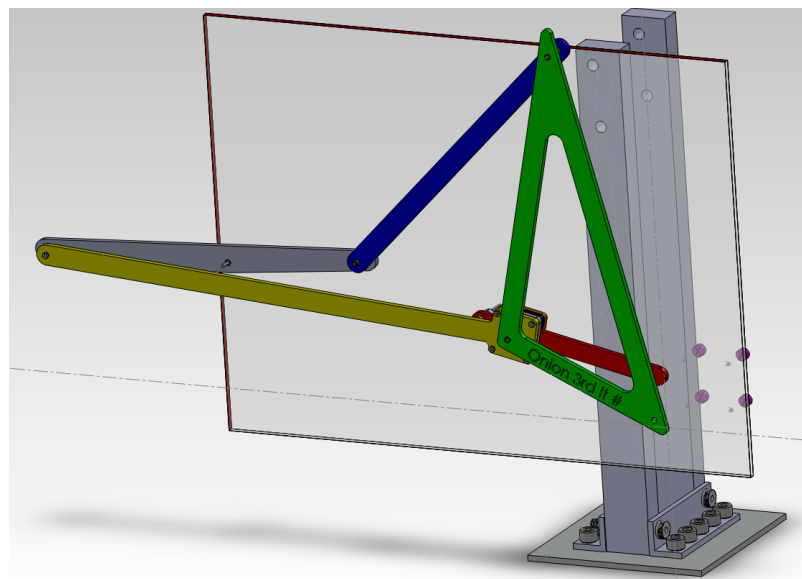
# Six-Bar Linkage to Guide Bolt Pattern Assembly

## Wheel Lug Nut Torque

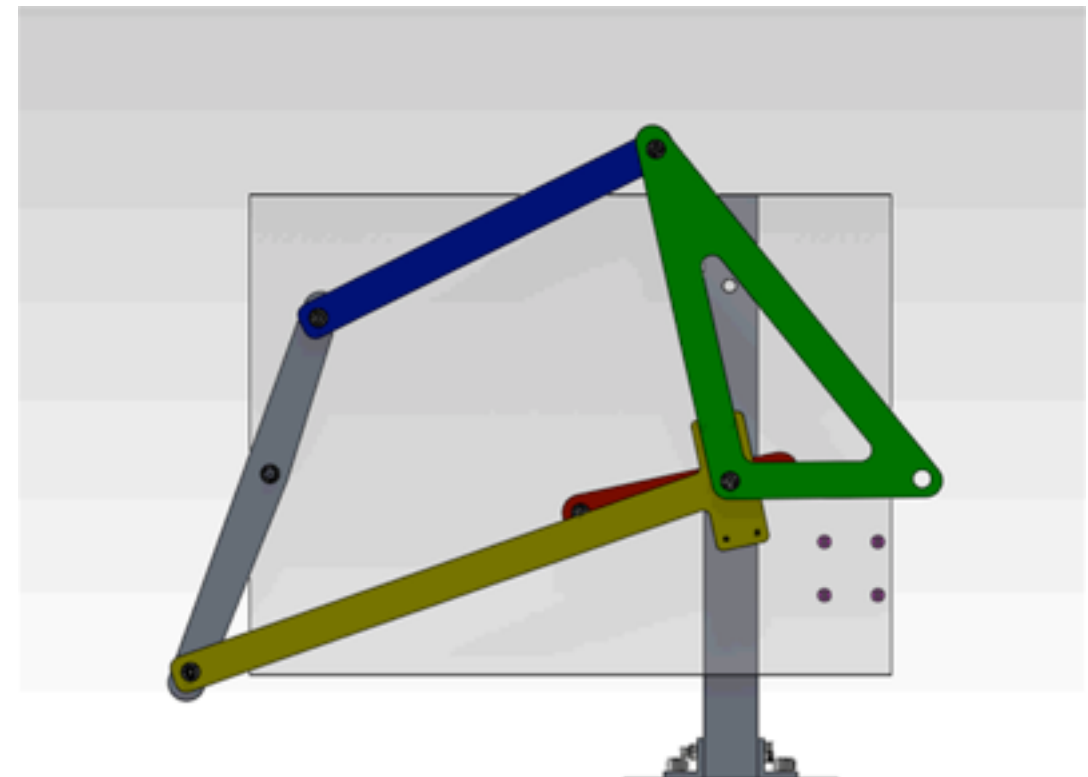


5,365,810 Hitachi Pneumatics

4,639,996 Xerox Motors



Six-bar linkage





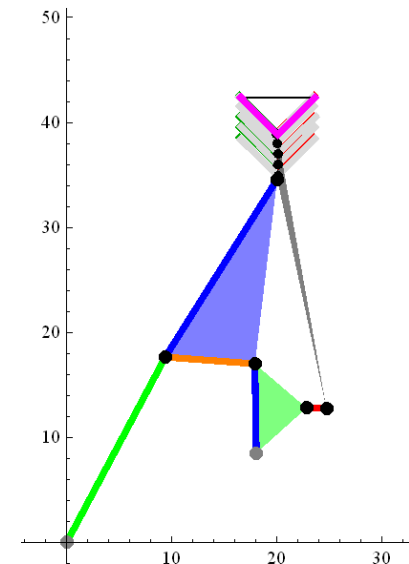
# Instrumentation for Spinal Surgery



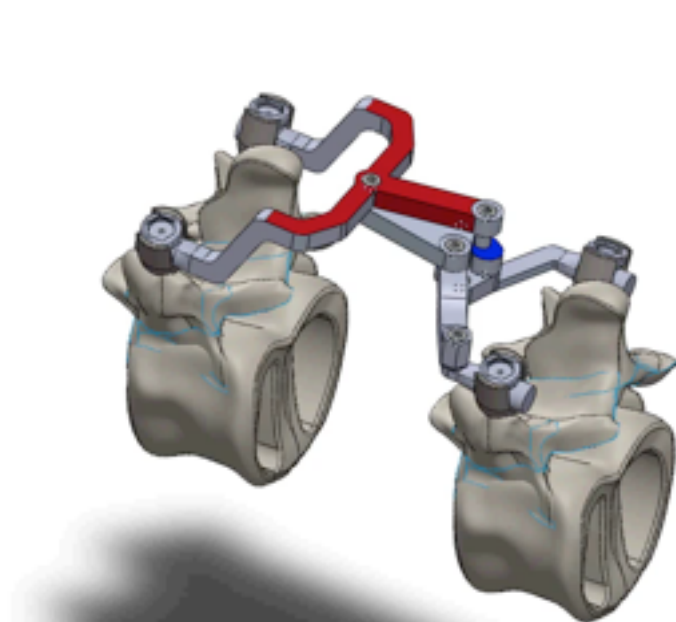
Spinal Implant



Distraction Tool



After 10,000 iterations yielded 54 useful linkages.



Adjustable Spinal Implant



Distraction Tool



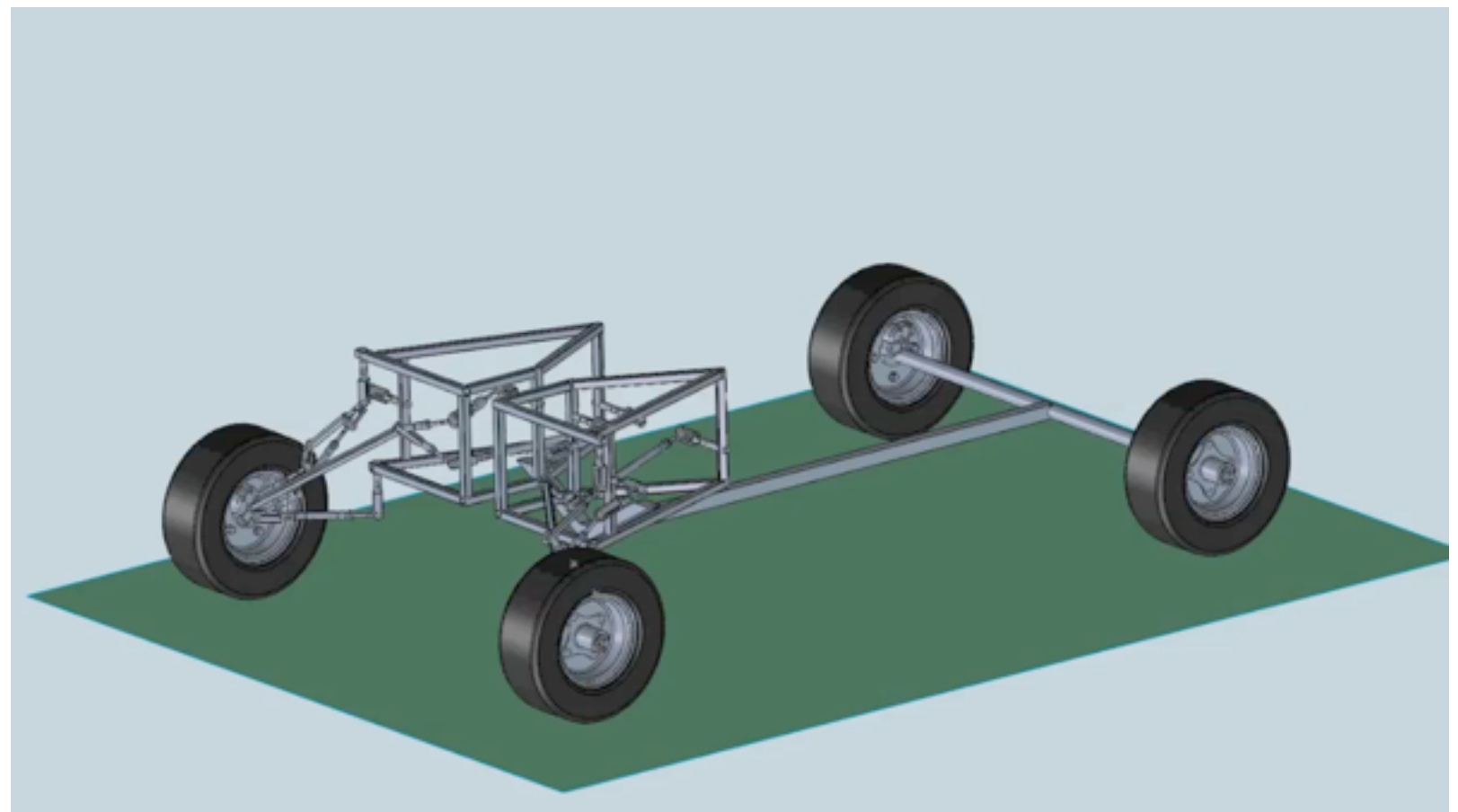
# Geometry Varying Steering Linkage

A steering linkage generally is a planar four-bar linkage designed to turn the inner wheel more than the outer wheel. This is known as Ackermann geometry.

This steering linkage is constructed from 2 5SS platform linkages.

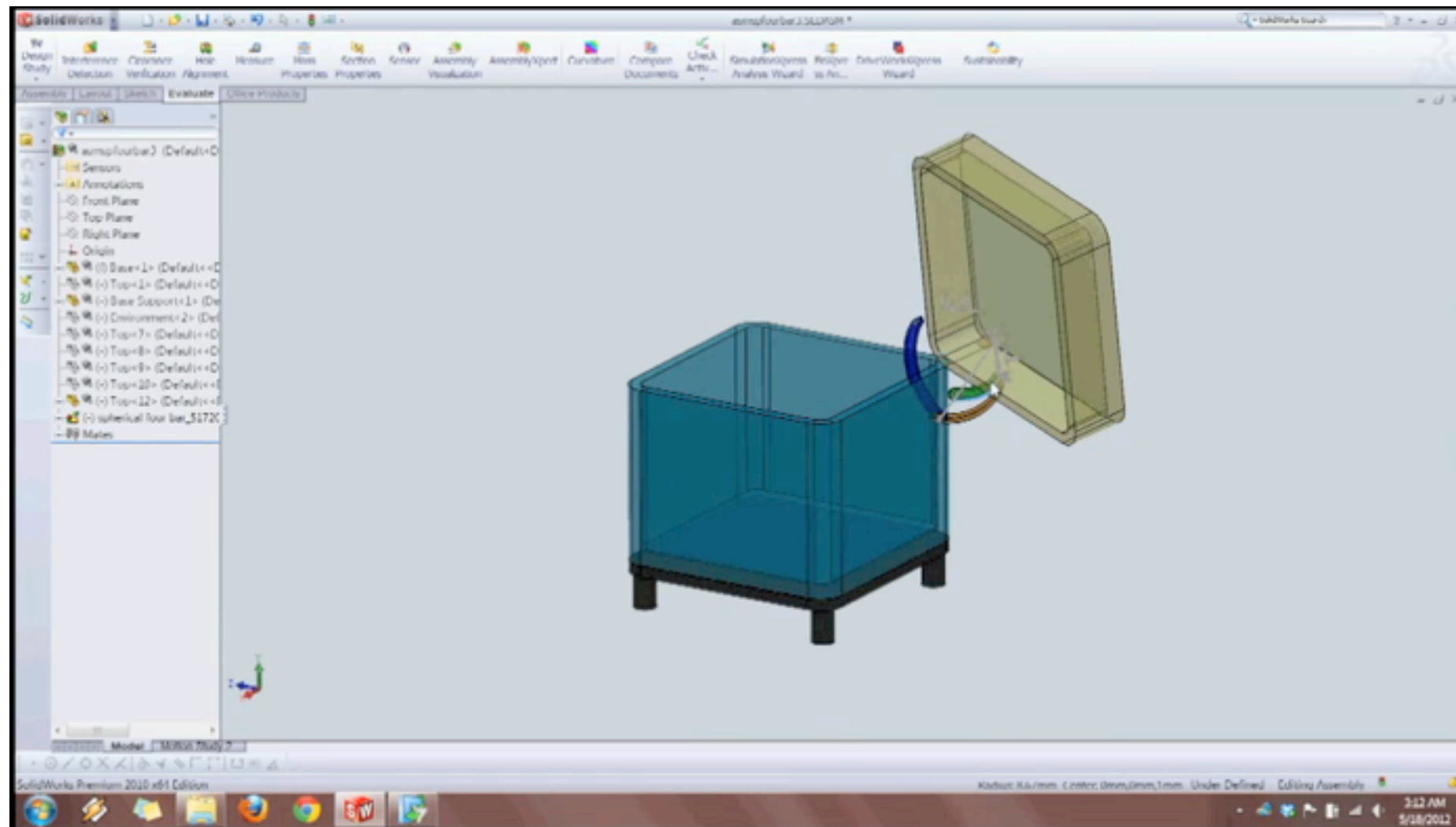
The geometry of this steering linkage adjusts to achieve:

1. Ackerman geometry,
2. Shorten wheelbase,
3. Increase front track,
4. Decrease front inner ride height,
5. Decrease front inner camber.

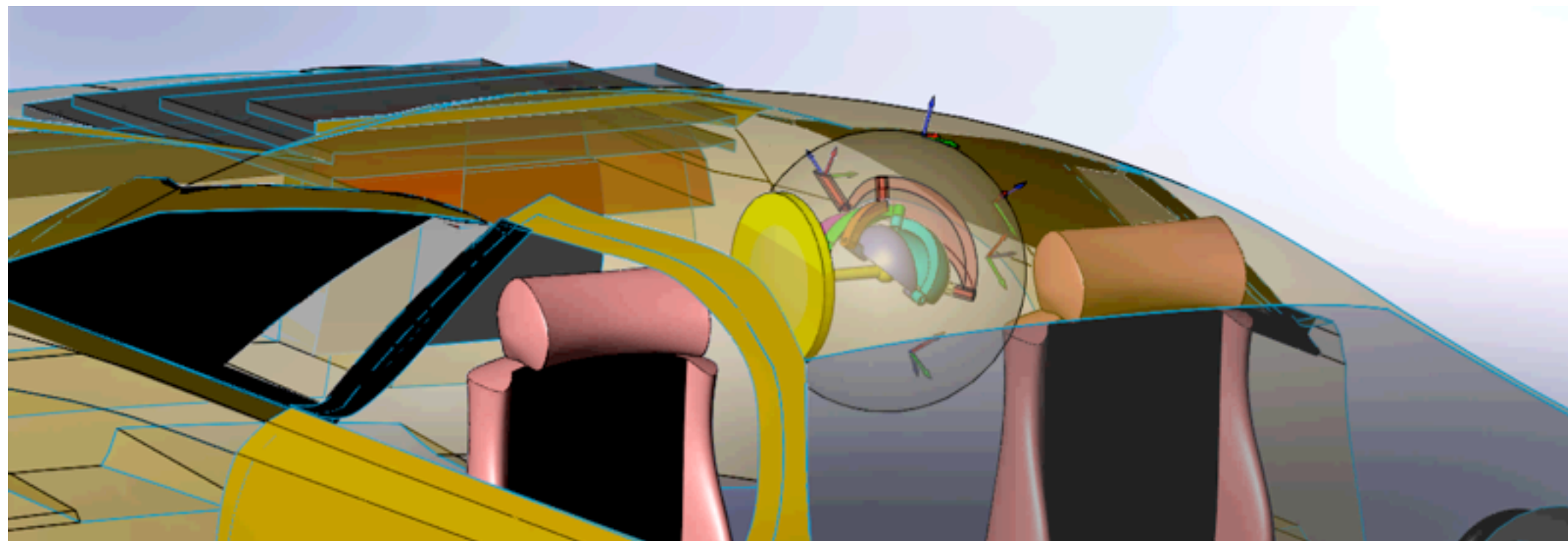
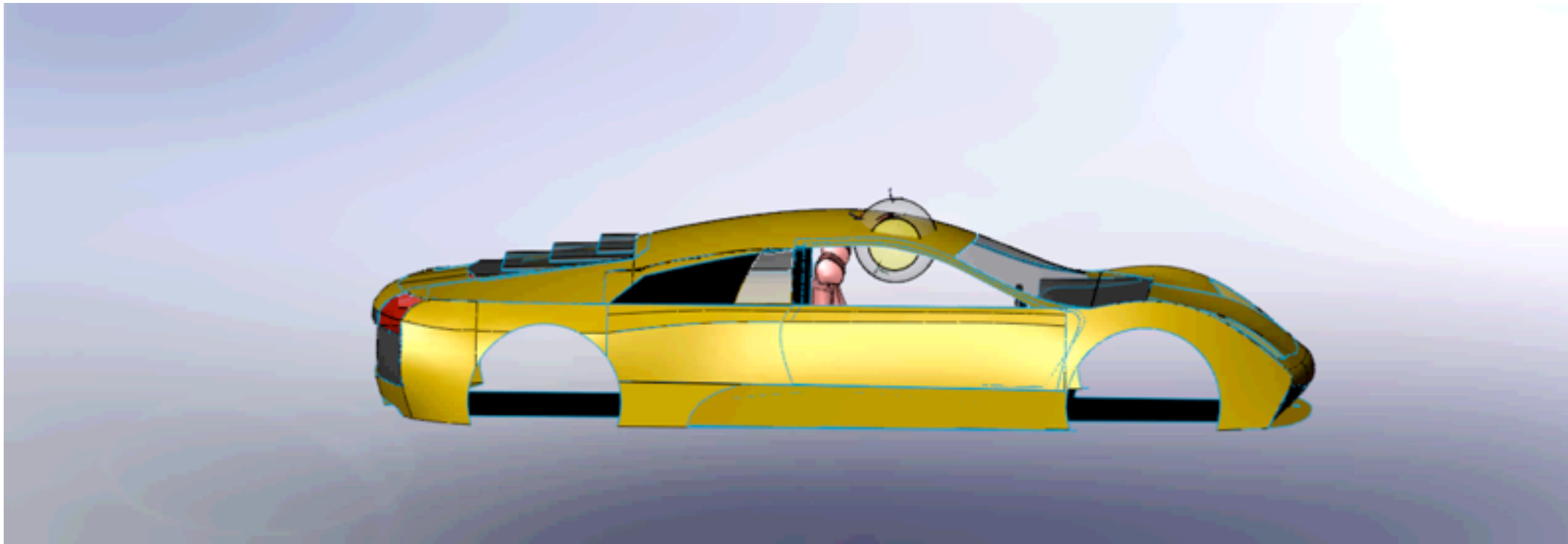


## 5. New Research

# Spherical Four-Bar Linkages: MechGen 4.0



# Spherical Six-Bar Linkage Car Door Concept





# Spherical Eight-Bar Car Door Concept



# Molecular Linkages

Markku J. Lampinen, Tuula Nojonen Electric dipole theory and thermodynamics of actomyosin molecular motor in muscle contraction, *Journal of Theoretical Biology* 236 (2005) 397–421

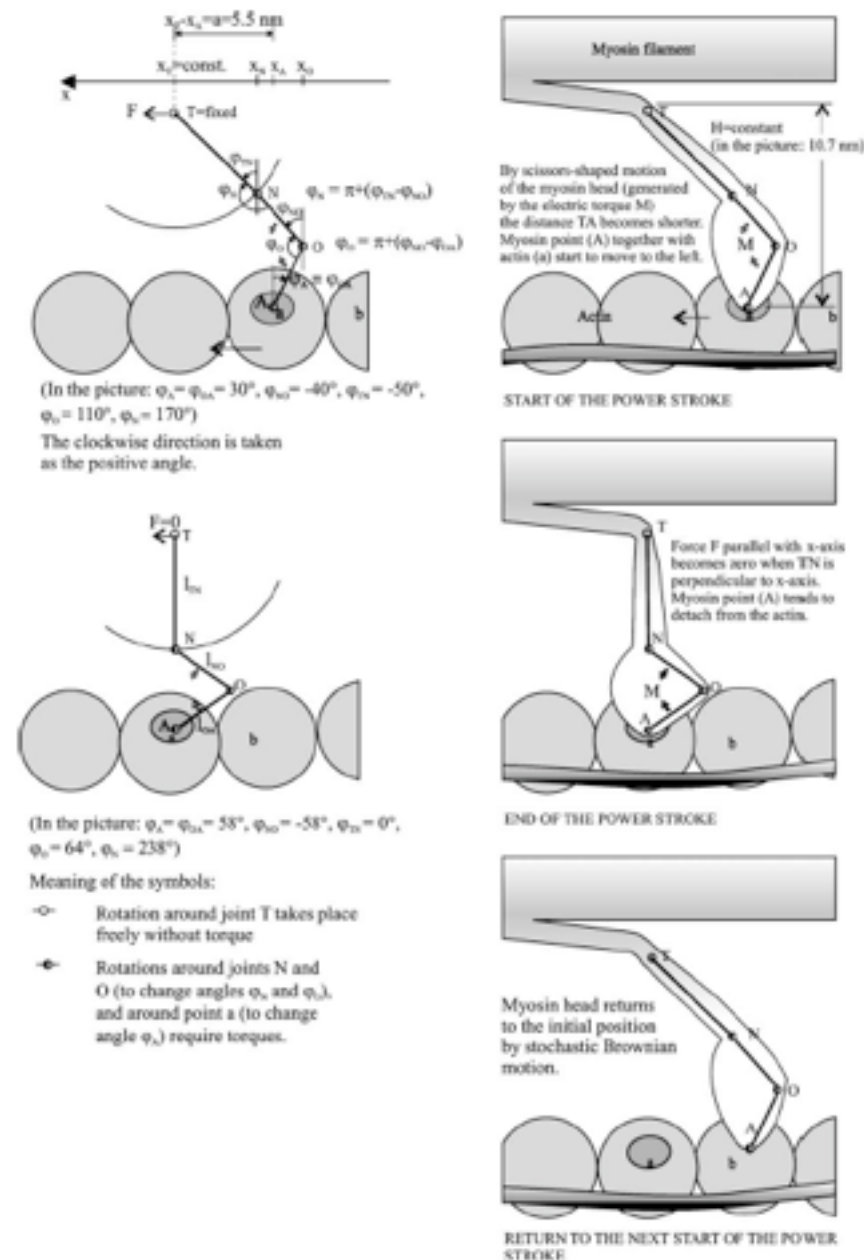
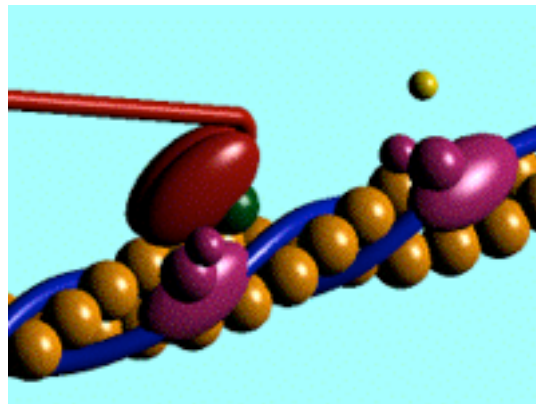
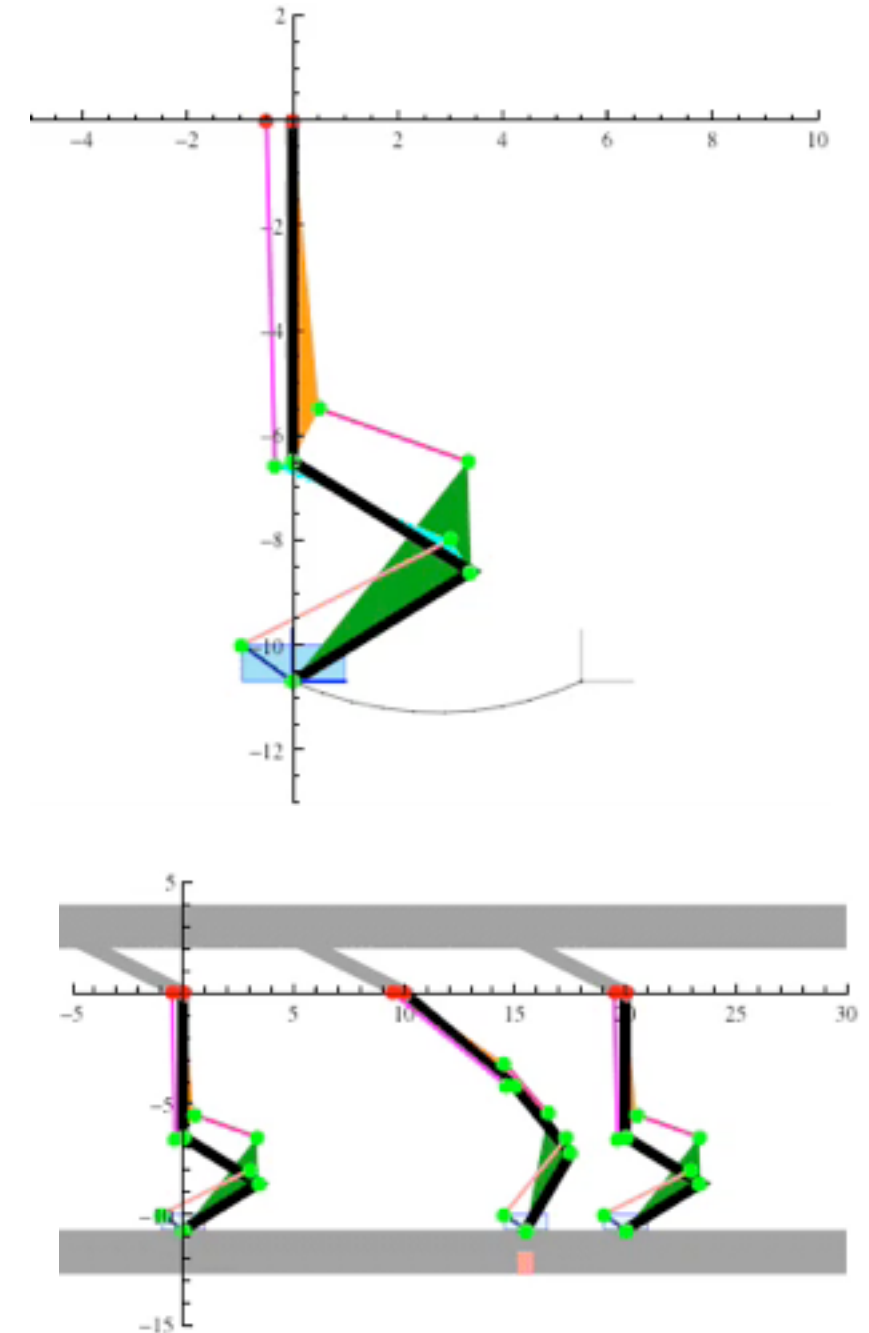
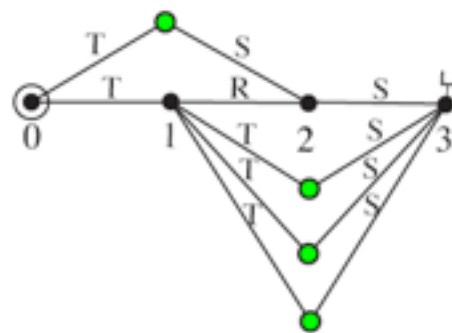
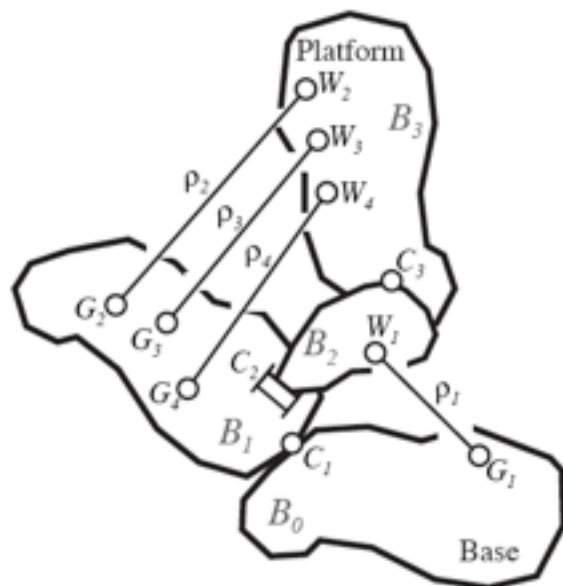


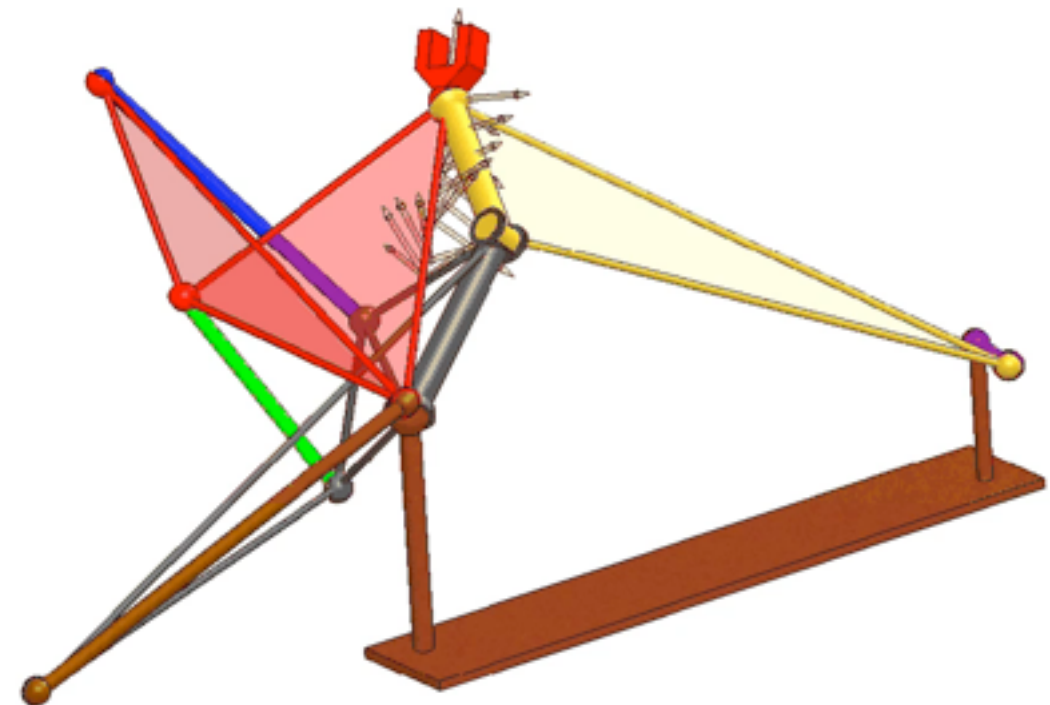
Fig. 2. Kinematics of scissors-shaped motion of the myosin head.



# Constrained TRS Robot

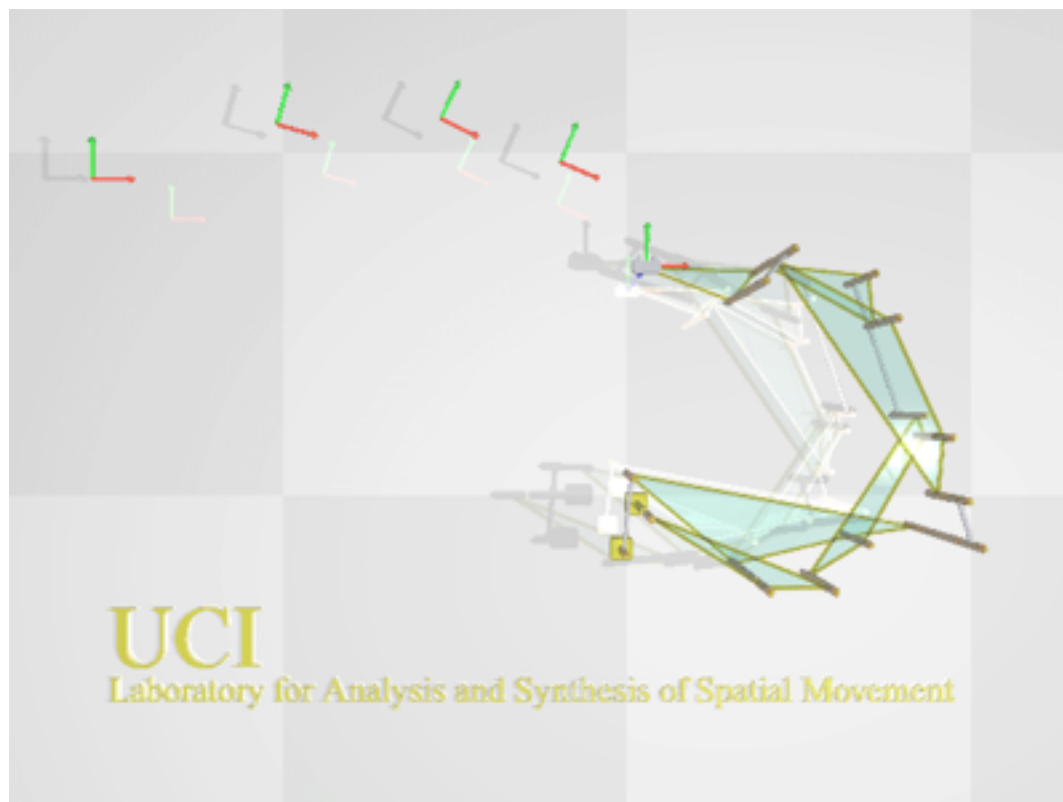


- Given a TRS robot, we can yield as many as 2700 two degree-of-freedom linkage if we only consider adding TS constraints.
- This could yield as many as over 4 million design candidates.
- The figure on the left shows one of the possible 2700 structure and the figure below shows a design candidate of this structure.



# Conclusions

- The computer aided invention of mechanical movement helps the designer match a desired task to a linkage system that yields a workable design
- Embedding this capability in geometric modeling software provides a new capability for invention.



**Our experience shows that this capability provides a new world of opportunities for the mechanism designer.**